THE FORMATION OF FARADAY WAVES ON A LIQUID COVERED WITH AN INSOLUBLE SURFACTANT: INFLUENCE OF THE SURFACE EQUATION OF STATE.

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Abstract— **In this work the effects of relatively large amounts of insoluble surfactants on the formation and evolution of two-dimensional Faraday waves is analyzed by means of numerical experiments. To describe the functional relationship between the surface tension and the local concentration of the absorbed solute, two equations of state are used: the Frumkin and the Langmuir expressions. A linear approximation is also employed for comparison. Results obtained show that the threshold conditions for the formation of the waves depend on the nature of the surfactant; nevertheless, the differences detected diminish as the amplitude of the external excitation is augmented.**

Keywords— **free-surface, surfactants, finite-element, Faraday waves.**

I. INTRODUCTION

When a container partially filled with a liquid is vertically vibrated, stationary waves with a frequency equal to one half the frequency of the external excitation can be observed at the liquid-air interface. These waves were reported by Faraday in 1831 but it was not until 1954 that the first theoretical analysis of this problem was presented by Benjamin and Ursell. These authors studied the stability of the interface of an ideal fluid in irrotational motion, for infinitesimally small amplitudes of the free surface oscillations. They derived an infinite set of Mathieu's equation and they showed that resonance is responsible for the wavy motion. They also concluded that the system is always unstable when the frequency of the external vibration is equal to twice the natural frequency of the system, even for forcing amplitudes infinitesimally small. This unrealistic result is a consequence of the ideal behavior assumed.

Since the pioneering work of Benjamin and Ursell (1954), many theoretical and experimental contributions dealing with different aspects of the problem have been published (for a review see Miles and Henderson 1990, Miles 1993, and Perlin and Schultz, 2000). Even though the damping effect on surface waves produced by surface active agents is well known since ancient times (Franklin, 1774), very few articles are concerned with the influence of surfactants on Faraday waves. Miles (1967) performed a weakly non linear analysis in which approximate analytical expressions for the damping coefficients of surface waves under the influence of surfactants were established. Henderson (1998) measured damping rates of the fundamental axisymmetric Faraday waves in a cylindrical container and compared the experimental values with those obtained evaluating the approximate analytical expression of Miles. She found a reasonable agreement between them even when the amplitude of the waves were large.

Kumar and Matar (2002a, b; 2004) presented three analyses about the role of insoluble surfactants on the critical oscillation amplitude required to form Faraday waves. In the first of these studies, they performed a linear stability analysis for fluids of arbitrary viscosity and depth. As a consequence of the assumptions introduced, a time-independent concentration of surfactant is the only solution compatible with the existence of Marangoni flows, this solution being possible in the limit of very high Péclet number. These authors proposed that the distribution of solute presents a spatial shift with the free surface deflections and they reported solutions using the spatial phase angle as an arbitrary parameter. The main conclusion reported is that the surfactant may either raise or lower the amplitude of the external oscillation needed to produce a wavy interface, depending on the value of the shift.

In the second article Kumar and Matar (2002b) established the magnitude of the minimum external force needed to form two-dimensional Faraday waves at the free surface of a liquid layer covered with an insoluble surfactant. The approach employed in this work is the lubrication approximation based on the assumption that the liquid thickness is very small compared to the wavelength of the disturbance. The results obtained show that the contaminated liquid layer becomes more stable as the elasticity number (i.e. the ratio between the Gibbs elasticity and the surface tension) increases; nevertheless, they predict that a clean system meeting the requirements of the approximations used can not be excited, a result that contradicts the predictions of the linear stability analysis (Kumar and Tuckerman, 1994).

In the third work, Kumar and Matar (2004) analyzed the formation of standing waves when the free surface is covered with an insoluble surfactant. They performed a full linear stability analysis of the problem in which surfactant convection is rigorously accounted for. They