A NEW REFINED THEORY OF PLATES WITH TRANSVERSE SHEAR DEFORMATION FOR MODERATELY THICK AND THICK PLATES

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Abstract — In this paper we propose a new refined shear deformation plate theory which possesses a series of desirable features, the most salient of which are as follows: (1) loads, which are generally applied on the middle surface of the plate, act on the upper surface of the plate; (2) equations applicable to the calculation of the stresses in isotropic plates which provide the same order of accuracy as several theories with second-order shear deformation effects; (3) a theory, in the classical sense defined, since it gives easy expressions for application to problems in different fields in architecture and civil engineering.

Keywords — Thick plates, first-order shear deformation theory, moderately thick plates.

I. INTRODUCTION

A rectangular plate is usually considered thin if its thickness is less than a tenth of the minor dimension. When this limitation is not fulfilled it is a moderately thick (a term introduced by Love (1944)) or thick plate.

Since Reissner (1945), Mindlin (1951), Hencky (1947) and Reissmann (1988) elaborated their technical calculation theories (first-order shear deformation theory), with the objective of widening the field of application of plates, many authors have studied this important field of structural mechanics.

Among other works, the current trend in the study of plates can be deduced from the themes of the articles collected by Voyiadjis and Karamanlidis (1990) and Kienzler et al. (2004). In the first publication it can be seen that four papers make a direct reference to Voyiadjis et al. (1981, 1987, 1990) and Baluch and Voyiadjis (1984) show different applications of this method to thick plates but we found no analytical solutions based on Fourier series.

As noted in this work, not many authors have extended these studies to include thick plates.

The works of Voyiadjis et al. (1981, 1987, 1990) and Baluch and Voyiadjis (1984) show different applications of this method to thick plates but we found no analytical solutions based on Fourier series.

Since the 1980s, many higher-order plate theories have been developed. Perhaps, the most popular third-order plate theory is Reddy’s (Reddy, 1990). We must also mention Touratier (1991) (transverse strain distribution as a sine function), Soldatos (1992) (hyperbolic shear deformation theory) and Shi (2007). Shi’s theory was similar to Reddy’s in the sense that he used a parabolic distribution of transverse shear strain.

All these theories are characterized by the need for a high level of mathematical complexity to obtain solutions. Furthermore, the problems which are solved analytically only constitute several specific examples.

In this paper, we deduced new equations for moderately thick and thick plates. Like Reddy (1990) and Shi (2007), we also propose a parabolic distribution of shear strain through the thickness. Unlike them, however, and taking into account the semi-inverse method, we start not with the displacement field but with the distribution of shear strains through the thickness. Other interesting features of the theory are presented below.

II. HYPOTHESIS AND OBTAINING GOVERNING EQUATIONS

The hypotheses to consider are as follows:

1) The loads, which are distributed, act at the upper surface of the plate in the downward z-direction and are perpendicular to the middle surface, and the displacements of the points located in the middle surface are also sensibly perpendicular to the middle surface (which is practically inelastic), that is, \( \hat{u} \approx 0 \) and \( \hat{\theta} \approx 0 \) where \( \hat{u} \) and \( \hat{\theta} \) are the displacements according to the x- and y-axes of the points located in the middle surface. The deflection \( w \) according to the z-axis of a generic point not located in the middle surface is given by

\[
w = \hat{w} + f(x),
\]

where \( \hat{w} \) is the deflection according to the z-axis of the points located in the middle surface. In the longitudinal deformation calculation subject to the thickness, Poisson’s effect is not considered; that is,

\[
\varepsilon_x \approx \frac{\varepsilon_y}{w} \approx 0
\]
2) The fibres pertaining to the plate, which are straight and perpendicular to the middle surface before the deformation, do not continue to be perpendicular, and they bend in such a way that the shearing strains are defined by a parabolic distribution throughout the thickness, given by

\[
\gamma_{xz} = \left(1 - \frac{2z^2}{h^2}\right) \tilde{\gamma}_{xz}; \quad \gamma_{yz} = \left(1 - \frac{2z^2}{h^2}\right) \tilde{\gamma}_{yz}
\]

where \(\tilde{\gamma}_{xz}\) and \(\tilde{\gamma}_{yz}\) are the shearing strains in the points of the middle surface.

3) The rotation \(w_{xy}\) of a differential element around the \(z\)-axis is null for all points of the plate, 

\[
w_{xy} = \frac{1}{2} \left( \frac{\partial \psi}{\partial y} - \frac{\partial \epsilon}{\partial y} \right) = 0.
\]

This important condition is deduced analytically if we establish the equilibrium of a plate element in its deformed configuration and take into consideration Reissner’s kinematic assumptions (Martínez Valle, 2012).

We denote by \(\tilde{\omega}\) the deflection, according to the \(z\)-axis, of the generic point \(m\) located on the plate’s middle-surface, \(\tilde{\theta}_x\) the angle rotated by the rectilinear segment normal to middle surface around the \(ox\)-axis, and \(\tilde{\theta}_y\) the angle rotated around the \(oy\)-axis. According to its definition, the shearing strain in the \(xz\)-surface at the point \(m\) located on the middle surface \((\tilde{\gamma}_{xz})\) is

\[
\tilde{\gamma}_{xz} = \frac{\partial \tilde{\omega}}{\partial y} - \frac{\partial \epsilon}{\partial y},
\]

which, as we can see in Fig. 1, is

\[
\tilde{\gamma}_{xz} = \tilde{\theta}_y + \frac{\partial \phi}{\partial x},
\]

Likewise, we deduce

\[
\tilde{\gamma}_{yz} = -\tilde{\theta}_x + \frac{\partial \phi}{\partial y},
\]

and we may also write

\[
\tilde{\gamma}_{xz} = \frac{\tilde{\gamma}_{xz}}{\tilde{\omega}} \quad \text{and} \quad \tilde{\gamma}_{yz} = \frac{\tilde{\gamma}_{yz}}{\tilde{\omega}}.
\]

where \(\tilde{\gamma}_{xz}\) and \(\tilde{\gamma}_{yz}\) are the shearing stresses at the points located on the middle surface.

Therefore shearing stresses at the points of the middle surface are

\[
\frac{\tilde{\gamma}_{xz}}{\tilde{\omega}} = \frac{\partial \phi}{\partial x}; \quad \frac{\tilde{\gamma}_{xz}}{\tilde{\omega}} = -\frac{\partial \epsilon}{\partial x}.
\]

The shearing strains at a generic point are

\[
\tilde{\gamma}_{xz} = \frac{\partial \phi}{\partial x} + \frac{\partial \wp}{\partial x} = \left(1 - \frac{4z^2}{h^2}\right) \tilde{\gamma}_{xz} ,
\]

\[
\tilde{\gamma}_{yz} = \frac{\partial \phi}{\partial x} + \frac{\partial \wp}{\partial y} = \left(1 - \frac{4z^2}{h^2}\right) \tilde{\gamma}_{yz},
\]

where

\[
\tilde{\omega} = \tilde{\omega}^\prime, \quad \tilde{\epsilon} = \tilde{\epsilon}^\prime.
\]

We may deduce the displacements of a generic point:

\[
u = \tilde{u} + \tilde{\theta}_y \cdot z - \frac{4z^2}{h^2} \tilde{\omega}^\prime, \quad \tilde{v} = \tilde{\theta}_x \cdot z - \frac{4z^2}{h^2} \tilde{\omega}^\prime.
\]

and from them we obtain the rotations around the \(x\)- and \(y\)-axes

\[
w_{x\tilde{x}} = -\frac{1}{2} \left( \frac{\partial \phi}{\partial y} - \frac{4z^2}{h^2} \tilde{\gamma}_{xz} \right),
\]

\[
w_{y\tilde{y}} = -\frac{1}{2} \left( \frac{\partial \phi}{\partial x} - \frac{4z^2}{h^2} \tilde{\gamma}_{yz} \right).
\]

Considering the third hypothesis and the value of the rotation around the \(z\)-axis at all points,

\[
w_{xy} = 0 = \tilde{\omega}_{xy} \left(1 - \frac{4z^2}{h^2}\right) \left( \frac{\partial \phi}{\partial y} + \frac{\partial \wp}{\partial y} \right),
\]

we deduce

\[
\frac{\partial \phi}{\partial y} + \frac{\partial \wp}{\partial y} = 0.
\]

The strains \(\epsilon_x\), \(\epsilon_y\), and \(\gamma_{xy}\) are

\[
\epsilon_x = \frac{\partial \psi}{\partial x} + \frac{\partial \wp}{\partial x} = \frac{4z^2}{3h^2} \gamma_{xz},
\]

\[
\epsilon_y = \frac{\partial \psi}{\partial y} + \frac{\partial \wp}{\partial y} = \frac{4z^2}{3h^2} \gamma_{yz},
\]

\[
\gamma_{xy} = \tilde{\gamma}_{xy} - \frac{\partial \phi}{\partial y} = \frac{4z^2}{3h^2} \gamma_{xz}.
\]

The shearing stresses \(\tau_{xz}\) and \(\tau_{yz}\) are deduced from the expressions given by Hooke’s law and Lamé’s equations:

\[
\tau_{xz} = \left(1 - \frac{4z^2}{h^2}\right) \tilde{\tau}_{xz},
\]

\[
\tau_{yz} = \left(1 - \frac{4z^2}{h^2}\right) \tilde{\tau}_{yz},
\]

The equilibrium equations of the plate element of differential sides are

\[
\frac{\partial \phi}{\partial x} + \frac{\partial \wp}{\partial y} + P = 0,
\]

\[
Q_{xx} = \frac{\partial \phi}{\partial x} + \frac{\partial \wp}{\partial y},
\]

\[
Q_{yz} = \frac{\partial \wp}{\partial x} + \frac{\partial \wp}{\partial y},
\]

The transverse shear forces in the faces of the plate element are

\[
Q_{xx} = \frac{1}{2} \int_0^h \tilde{\tau}_{xz} dz = \frac{1}{2} \int_0^h \left(1 - \frac{4z^2}{h^2}\right) \tilde{\tau}_{xz} dz = \frac{2h}{3} \tilde{\tau}_{xz},
\]

\[
Q_{yz} = \frac{1}{2} \int_0^h \tilde{\tau}_{yz} dz = \frac{1}{2} \int_0^h \left(1 - \frac{4z^2}{h^2}\right) \tilde{\tau}_{yz} dz = \frac{2h}{3} \tilde{\tau}_{yz},
\]

and substituting in Eq.21 we obtain

\[
\frac{\partial \phi}{\partial x} + \frac{\partial \wp}{\partial y} = -\frac{3P}{2h}.
\]

And also after substituting \(\tilde{\tau}_{xz}\) and \(\tilde{\tau}_{yz}\) with their values,

\[
\frac{\partial \wp}{\partial x} + \frac{\partial \wp}{\partial y} = -\frac{3P}{2h}.
\]

The normal stress \(\sigma_z\) is determined by means of the following equation of the internal equilibrium of the elasticity:

\[
\frac{\partial \phi}{\partial x} + \frac{\partial \wp}{\partial y} + \Delta \psi = 0,
\]

from which we deduce

\[
\frac{\partial \wp}{\partial x} + \frac{\partial \wp}{\partial y} = \frac{1}{2} \left(1 - \frac{4z^2}{h^2}\right) \frac{\partial \phi}{\partial y} + \frac{\partial \wp}{\partial y} = \frac{3P}{2h} \left(1 - \frac{4z^2}{h^2}\right).
\]

Integrating throughout the thickness between a generic point located at a height \(z\) and the upper surface, in which \(\sigma_z\) is equal to \(P\), we get

\[
P = \frac{3P}{2h} \left(1 - \frac{4z^2}{h^2}\right) dz = \frac{3P}{2h} \left(\frac{h^3}{3} - z + \frac{4z^2}{3h^2}\right).
\]

Thus we deduce
In the presented theory, observing the fulfillment of the Equations \( \sigma_x \) appears in Kromm’s refined plate theory (Panc, 1975).

The system of governing equations turns out to be disconnected, that is, to separate the calculation of the moment stress resultants by

\[
M_x = D \left( \frac{\partial^2 \varphi}{\partial y^2} + \mu \frac{\partial^2 \varphi}{\partial x \partial y} \right) - \frac{D}{\hbar^4} \left( \frac{\partial^4 \varphi}{\partial x^4} + \mu \frac{\partial^4 \varphi}{\partial x^2 \partial y^2} \right) + \frac{\mu \hbar^2}{10(1-\mu)} \cdot \frac{\partial^2 \varphi}{\partial x \partial y} \cdot (31)
\]

The differential equations required to determine \( \vartheta_x, \vartheta_y \) and \( \varpi \) may be obtained by applying the principle of minimum energy and making null the first variation of the total potential energy or alternatively by deciding to apply variational formulation or to raise the static equilibrium equations of the plate element of differential sides. Taking the last method, we substitute the moments calculated in the last two equations

\[
\Delta \vartheta_x + \frac{1}{2} \frac{\partial}{\partial x} \left( \frac{\partial^2 \varphi}{\partial y^2} \right) + 1 \frac{\partial}{\partial x} (\Delta \varpi) = \frac{5(1-\mu)}{h^2} \left( \vartheta_x + \varpi \right)
\]

III. GOVERNING EQUATIONS DISCONNECTED FOR DISPLACEMENTS AND ROTATIONS

The system formed by Eqs. 27, 51 and 52 may be disconnected, that is, to separate the calculation of displacements from the calculation of rotations. If we transform it applying Laplace’s operator to Eq. 27:

\[
\Delta \left( \frac{\partial^2 \varphi}{\partial y^2} + \mu \frac{\partial^2 \varphi}{\partial x \partial y} \right) + \mu \hbar^2 \frac{\partial^2 \varphi}{\partial x \partial y} = \frac{3}{2} \frac{\partial^2 \varphi}{\partial x \partial y} \cdot \Delta \varphi
\]

However, Eqs. 51 and 52 yield

\[
\frac{\partial}{\partial x} (\Delta \vartheta_x) - \frac{\partial}{\partial y} (\Delta \vartheta_y) = \frac{1}{4} \frac{\partial^2}{\partial x^2} (\Delta \varphi) + \frac{1}{4} \frac{\partial^2}{\partial y^2} (\Delta \varphi) + \frac{1}{4} \frac{\partial^2}{\partial x \partial y} (\Delta \varphi) + \frac{5(1-\mu)}{h^2} \left( \vartheta_x + \varpi \right)
\]

According to Eq. 27 the last equation can be written

\[
\frac{\partial}{\partial x} (\Delta \vartheta_x) - \frac{\partial}{\partial y} (\Delta \vartheta_y) = \frac{1}{4} \Delta \varphi - \frac{5(1-\mu)}{h^2} \frac{3}{4} P - \frac{\mu \hbar^2}{6(1-\mu)} \Delta P
\]

and substituting in Eq. 53 we obtain

\[
\frac{\partial}{\partial x} (\Delta \vartheta_x) - \frac{\partial}{\partial y} (\Delta \vartheta_y) = \frac{1}{4} \Delta \varphi - \frac{5(1-\mu)}{h^2} \frac{3}{4} P - \frac{\mu \hbar^2}{6(1-\mu)} \Delta P + \frac{\Delta \varpi}{\partial x} = \frac{3}{5(1-\mu)} \frac{\Delta \varphi}{\partial x} \cdot \Delta P
\]

The system of governing equations turns out to be

\[
\Delta \varphi = \frac{P}{\partial P} + \frac{6(1-\mu)}{5(1-\mu)} \Delta \varphi \cdot \Delta P
\]
\[ \Delta \theta_x + \frac{5(1-\mu)}{h^2} \theta_x = -\frac{1}{4} \frac{\partial}{\partial y} (\Delta \omega) + \frac{5(1-\mu)}{h^2} \frac{\partial \psi}{\partial y} + \frac{\mu h^2}{8h(1-\mu)} \frac{\partial^2 \omega}{\partial y^2} + \frac{\mu h^2}{8h(1-\mu)} \frac{\partial^2 \psi}{\partial y^2} \]  
(59)

\[ \Delta \theta_y + \frac{5(1-\mu)}{h^2} \theta_y = -\frac{1}{4} \frac{\partial}{\partial x} (\Delta \omega) + \frac{5(1-\mu)}{h^2} \frac{\partial \psi}{\partial x} - \frac{\mu h^2}{8h(1-\mu)} \frac{\partial^2 \omega}{\partial x^2} - \frac{\mu h^2}{8h(1-\mu)} \frac{\partial^2 \psi}{\partial x^2} \]  
(60)

V. DISCUSSION OF THE EQUATIONS DEDUCED

If we analyze the system of Eqs. 58 to 60, we can see that the term \(-\frac{1}{4} \frac{\partial}{\partial y} (\Delta \omega)\) appears as a consequence of assumption 2 of this work (parabolic distribution of the transverse shear strains through the thickness of the plate).

The terms \(-\frac{\mu h^2}{8h(1-\mu)} \frac{\partial^2 \psi}{\partial y^2}\) and \(-\frac{\mu h^2}{8h(1-\mu)} \frac{\partial^2 \omega}{\partial x^2}\) are a consequence of the first assumption, which is closer to the real physical problem.

Therefore, the equations proposed are modified expressions of a third order plate theory taking into account the important condition exposed in assumption 3.

If we neglect these terms, Eqs. 58 to 60 are a close variant of the ones obtained in the Vlasov theory.

In addition, as we have pointed out, the value of the stress \(\sigma_x\) in this work is identical to the one proposed in the theory of plates by Kromm (1953). Similar results have been found in Reddy’s work (Reddy, 1999).

Lastly, Eq. 67 has the same structure as the one proposed by Reissman (1988), relating displacements and moment-sum, which is

\[ \Delta W = \frac{M}{D} - \frac{P}{K E H} \]  
(74)

It only differs from Eq.74 in the term \((1 + \mu)\).

VI. ILLUSTRATIVE EXAMPLES

Example 1: Static analysis. General Solution and study of simply supported thick plates.

If we use Fourier series to express the deflections and rotations in the form,

\[ \begin{align*} 
q &= \sum \frac{a_n(y)}{\sin \frac{m \pi x}{a}} \\
\theta_x &= \sum \frac{b_n(y)}{\sin \frac{m \pi x}{a}} \\
\theta_y &= \sum \frac{c_n(y)}{\sin \frac{m \pi x}{a}} \\
\end{align*} \]  
(75)

\[ \begin{align*} 
w &= \sum \frac{w_n(y)}{\sin \frac{m \pi x}{a}} \\
\psi &= \sum \frac{\psi_n(y)}{\sin \frac{m \pi x}{a}} \\
\end{align*} \]  
(76)

The general solution of the system of Eqs. 58 to 60 can be deduced by solving the following system of differential equations,

\[ \begin{align*} 
\frac{m \pi a}{\alpha} w_n(y) + w''''(y) - 2 \left( \frac{m \pi a}{\alpha} \right)^2 w_n(y) + \left[ \frac{m \pi a}{\alpha} \right]^2 \omega_n(y) &= \frac{\alpha}{\sin (\frac{m \pi x}{a})} \left( \frac{\omega_n(y) - \omega_n(y)}{\sin (\frac{m \pi x}{a})} \right) \\
- \left[ \frac{m \pi a}{\alpha} \right]^2 + \left[ \frac{5(1-\mu)}{h^2} \right] T_x(y) + \frac{1-\mu}{2} T''(y) - \frac{5(1-\mu)}{h^2} \left[ \frac{m \pi a}{\alpha} \right]^2 \psi_n(y) &= - \frac{1}{4} \left( \frac{m \pi a}{\alpha} \right)^2 T''(y) + \frac{1-\mu}{2} T''(y) - \frac{5(1-\mu)}{h^2} \left[ \frac{m \pi a}{\alpha} \right] \frac{\psi_n(y)}{\sin \left( \frac{m \pi x}{a} \right)} \\
- \left[ \frac{m \pi a}{\alpha} \right]^2 w_n(y) - w''''(y) &= \frac{\mu h^2}{8h(1-\mu)} \psi_n(y) \\
- \frac{1}{4} \left( \frac{m \pi a}{\alpha} \right)^2 T''(y) + \frac{3}{4} T''(y) - \frac{5(1-\mu)}{h^2} \frac{\psi_n(y)}{\sin \left( \frac{m \pi x}{a} \right)} &= \frac{\mu h^2}{8h(1-\mu)} \psi_n(y) \\
\end{align*} \]  
(79)
\[
\begin{align*}
\psi &= \frac{q_0}{\pi^2 E} \left[ \frac{1}{(1 + \frac{a^2}{b^2}) - 6(1 - \mu)} \right] \\
\text{Substituting this coefficient in the displacement field (Eq. 75)}
\end{align*}
\]

\[
\psi = \frac{q_0 a^4}{D\pi^4 \left( 1 + \frac{a^2}{b^2} \right)^2} \left[ \frac{1}{(1 + \frac{a^2}{b^2} - 6(1 - \mu))} \right]
\]

which can be reduced to the expression obtained by Timoshenko and Woinowski (1959) in the case of \( t \rightarrow 0 \) and coincides exactly with the expression deduced with the higher-order plate theory of Vlasov (1958).

In addition to the advantages found for bending moments, major improvements have been found by calculating horizontal bending stresses \( \sigma_x \).

In Reissner's theory, for \( a = b = 3h \) and \( v = 0.3 \), the maximum horizontal bending stress is,

\[
\sigma_x(x, y, -h/2) = -1.78 \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}
\]

The exact solution with the theory of elasticity is,

\[
\sigma_x(x, y, -h/2) = -2.12 \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}
\]

In this work, according to Eq. 36, we found

\[
\sigma_x(x, y, -h/2) = -2.01 \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}
\]

reducing the error to 6%, approximately.

**Example 2**: Free vibration of a simply supported isotropic rectangular plate.

Applying the principle of d'Alembert to establish dynamic equilibrium equations for the study of transverse oscillations of plates, we need only consider in the equilibrium Eq. (57) the inertia forces rather than static load \( P \).

The problem to be solved is,

\[
\Delta \omega = -\frac{r y}{D} \ddot{w} - \frac{6(1+\mu)(a-2)}{5E} \Delta \dot{w}
\]

If the deflection is expressed as,

\[
w = [C_1 \cos(f t) + C_2 \sin(f t)] U(x, y)
\]

where \( f \) is the frequency.

Substituting,

\[
\Delta \omega = \frac{r y}{D} f^2 U + \frac{6(1+\mu)(a-2) y}{5E} f^2 \Delta U
\]

Boundary conditions are satisfied if we take the solution in the form,

\[
U_{mn} = \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b}
\]

Substituting again,

\[
\left( \frac{m \pi}{a} \right)^4 + \frac{2}{a b} (m \pi) + \left( \frac{n \pi}{b} \right)^4 - \frac{r y}{D} f^2 f_m^2 + \frac{6(1+\mu)(a-2) y}{5E} f_m^2 \left( \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2 \right) = 0
\]

and solving,

\[
f_m = \frac{\pi^2}{b^2} \left( n^2 + \left( \frac{b}{a} \right)^2 \right)
\]
Table 2. Parameter $\lambda$ of a simply supported plate, $\frac{a}{b} = 0.4$, $v = 0.3$

<table>
<thead>
<tr>
<th>Mode</th>
<th>h/a=0.01</th>
<th>h/a=0.1</th>
<th>h/a=0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.250</td>
<td>6.479</td>
<td>5.183</td>
</tr>
<tr>
<td>4</td>
<td>22.231</td>
<td>16.847</td>
<td>11.487</td>
</tr>
<tr>
<td>8</td>
<td>33.999</td>
<td>23.400</td>
<td>15.036</td>
</tr>
<tr>
<td>Sol.1(Liew)</td>
<td>7.250</td>
<td>6.477</td>
<td>5.183</td>
</tr>
<tr>
<td>Sol.4(Liew)</td>
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<td>16.845</td>
<td>11.487</td>
</tr>
<tr>
<td>Sol.8(Liew)</td>
<td>33.998</td>
<td>23.399</td>
<td>15.034</td>
</tr>
</tbody>
</table>

\[ \sqrt{\frac{Et^2}{(1+\mu)\gamma}} = \frac{1}{12(1-\mu) - 1,2(\mu - 2)\frac{a^2}{b^2}\left[\left(\frac{b}{a}\right)^2 + \pi^2\right]} \]

The first line of this equation corresponds to the classical solution (Leissa, 1973).

In order to compare the results, we consider a plate with $\frac{a}{b} = 0.4$, and the paper of Liew et al. (1993) in which the pb-2 Rayleigh-Ritz method was adopted. These results are referred to the nondimensional frequency parameter $\lambda$ given by,

\[ \lambda = \frac{f^2a^2}{\pi^2} \frac{\rho h}{D} \]  

(99)

Results are shown for different ratios of thickness and length (0.01, 0.1 and 0.2) in the following Table 2. When the ratio h/a increases, the differences with respect to the classical theory (Reissner) are greater and the results are closer to the work of Liew (1993).

VII. CONCLUSIONS

We have achieved the system formed by Eqs. 58, 59, 60, and 73, which have the same order of refinement as the one presented by Muhammad et al. (1990) for thick plates but with the advantage of following the exposition of the classical technical theories; it constitutes a more refined generalization than the presented one by Reissner (1988) for thick plates and that presented by Timoshenko and Woinowski (1959).

Also, assuming that the plane of application of the load is the upper plane of the plate, the first predictions of normal stress depending on the thickness values match those deduced by Kromm’s theory (Kromm, 1953).

General analytical solutions to this system of equations have been presented and excellent results for simply supported plates in static and dynamic analysis have been found. In future work we will present more complex analytical solutions for other boundary conditions, and numerical results for those cases in which analytical solutions are not possible.

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