

NECESSARY CONDITION ON THE ONSET OF DOUBLE-DIFFUSIVE CONVECTION IN COUPLE-STRESS FLUID IN HYDROMAGNETICS SATURATED BY A POROUS MEDIUM

G.C. RANA[†] and R.C. THAKUR[‡]

[†] Department of Mathematics, NSCBM Govt. P. G. College, Hamirpur-177 005, Himachal Pradesh, INDIA

E. Mail: drgcrana15@gmail.com

[‡] Department of Mathematics, Govt. P. G. College, Dhaliara-177 103, Himachal Pradesh, INDIA

Abstract— In this paper, the effect of magnetic field on double-diffusive convection in couple-stress fluid saturating a porous medium is considered. By applying linear stability theory and normal mode analysis method, a necessary condition is derived which states that the viscoelastic double-diffusive convection at marginal state, cannot manifest as stationary convection if the thermal Rayleigh number R , the medium permeability parameter P_l the couple-stress parameter F , the stable solute gradient S , medium porosity ε and the Chandrasekhar number Q , satisfy the inequality

$$R \leq \frac{4\pi^2}{P_l} \left[1 + 2\pi^2 F + \frac{P_l Q}{2\varepsilon} + \frac{P_l S a^2}{4\pi^4} \right]$$

the result clearly established the stabilizing character of couple-stress parameter, stable solute gradient and magnetic field whereas destabilizing character of medium permeability and porosity.

Keywords— Couple-Stress fluid, Double-diffusive convection, Magnetic field, Porous medium.

I. INTRODUCTION

The problem of double-diffusive convection in porous media has attracted considerable interest during the last few decades, because it has various applications in geophysics, food processing, soil sciences, ground water hydrology and nuclear reactors etc. A detailed account of the thermal convection of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar (1981). Lapwood (1948) has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding (1960).

In all the above studies, the fluid is considered to be Newtonian. Although the problem of thermal convection has been extensively investigated for Newtonian fluids, relatively little attention has been devoted to this problem with non-Newtonian fluids. With the growing importance of non-Newtonian fluids with magnetic field in modern technology and industries, the investigations on such fluids are desirable. One such type of fluid is couple-stress fluid. Stokes (1966) proposed and postulated the theory of couple-stress fluid. One of the applications of couple-stress fluid is its use to the study of the mechanism of lubrication of synovial joints, which

has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as lubricant. When fluid film is generated, squeeze film action is capable of providing considerable protection to the cartilage surface. The shoulder, knee, hip and ankle joints are the loaded-bearing synovial joints of human body and these joints have low-friction coefficient and negligible wear. Normal synovial fluid is clear or yellowish and is a viscous, non-Newtonian fluid.

According to the theory of Stokes (1966), couple-stresses are found to appear in noticeable magnitude in fluids. Since the long chain hylauronic acid molecules are found as additives in synovial fluid. Walicki and Walicka (1999) modeled synovial fluid as couple-stress fluid in human joints. Sharma and Thakur (2000) have studied the couple-stress fluid heated from below in hydromagnetics.

A good account of convection problems in a porous medium is given by Vafai and Hadim (2000), Ingham and Pop (1981) and Nield and Bejan (2006). Sharma and Rana (2001) have studied thermal instability of an incompressible Walters' (model B') elastico-viscous in the presence of variable gravity field and rotation in porous medium whereas stability of incompressible Rivlin-Ericksen elastico-viscous superposed fluids in the presence of uniform horizontal magnetic field in porous medium studied by Rana *et al.* (2011). Recently, Kumar (2011) studied stability of stratified couple-stress dusty fluid in the presence of magnetic field through porous medium whereas Rana and Sharma (2011) studied the hydromagnetic thermosolutal instability of compressible Walters' (model B') rotating fluid permeated with suspended particles in porous medium and found that magnetic field completely stabilizes the system.

Keeping in mind the importance in various applications mentioned above, our main aim in the present paper is to study the effect of magnetic field on double-diffusive convection in couple-stress elastico-viscous fluid in a porous medium.

II. MATHEMATICAL MODEL AND PERTURBATION EQUATIONS

Here, we consider an infinite, horizontal, incompressible couple-stress viscoelastic fluid of depth d , bounded by the planes $z=0$ and $z=d$ in an isotropic and homogeneous medium of porosity ε and permeability k_1 , which is acted upon by gravity $g(0,0,-g)$ and uniform vertical