A NEW FORMULATION TO THE SHORTEST PATH PROBLEM WITH TIME WINDOWS AND CAPACITY CONSTRAINTS

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***Abstract*— The Shortest-path problem with time-windows and capacity constraints (SPPTWCC) is a problem used for solving vehicle-routing and crew-scheduling applications. The SPPTWCC occurs as a sub-problem used to implicitly generate the set of all feasible routes and schedules in the column-generation formulation of the vehicle routing problem with time windows (VRPTW) and its variations. The problem is NP-hard in the strong sense. Classical solution approaches are based on a non-elementary shortest-path problem with resource constraints using dynamic-programming labeling algorithms. In this way, numerous label-setting algorithms have been developed. Contrarily to this approach and with the aim to obtain elemental and optimal solutions, we propose a new mixed integer-linear formulation to the SPPTWCC. Some valid inequalities that can be used to strengthen the linear relaxation of the SPPTWCC are also proposed. Numerical experiments on some VRPTW instances taken from Solomon's benchmark problems show that (near) optimal solutions are easily obtained in spite of the considerable problem size. Also the number of generated columns is kept at a very low level.**

***Keywords*— shortest path problem, MILP formulation, column generation, vehicle routing.**

I. INTRODUCTION

The shortest-path problem with time-windows and capacity constraints is a problem widely used for formulating vehicle-routing and crew-scheduling applications (Desaulniers *et al.*, 1998). The SPPTWCC consists of finding the shortest path from a source-node to some nodes of a network (while fulfilling timing and capacity constraints) that ends in a sink node. The term "shortest path" should be carefully interpreted: given costs associated to arcs and prices associated to the nodes, the aim is to find the least-cost path from the source node to the sink node. The SPPTWCC occurs as a sub-problem used to implicitly generate the set of all feasible routes in the column-generation formulation of the vehicle routing problem with time windows (VRPTW) and its variants (Cordeau *et al.*, 2002). It is NP-hard in the strong sense. In the VRPTW, the source and sink nodes are usually located in the same place. This place is commonly named "the depot". For n-depot routing problems, n source/sink pairs placed in the same location are usually defined. We may relax the problem to consider variants with source and sink nodes placed on different locations. This work deals just with the single-depot case but the more general cases are straight forward.

The SPPTWCC is also a problem with an economic meaning. I.e. given a set of profits associated to the nodes we aim to choose, at a non-zero cost, a subset of nodes that maximizes our net profit.

Classical solution approaches are based on the non-elementary shortest-path problem with resource constraints, using pseudo-polynomial dynamic programming labeling algorithms. Very refined and complex algorithms of this type have been developed. (See e.g. Houck *et al.,* 1980; Irnich and Desaulniers, 2005 and Irnich and Villenueve, 2006). These algorithms are very effective in generating, in addition to the best route, many solutions per iteration. On the contrary, our purpose is just to obtain the optimal solution to the SPPTWCC. Consequently, we propose a new mixed integer-linear formulation (MILP) to the problem.

This work is organized as follows: Section 2 describes the problem and presents its conventional MILP formulation. Section 3 presents a novel MILP formulation based on global precedence relationships. Advantages and weaknesses of this model are also discussed in this section. In Section 4 several preprocessing and polyhedral techniques are applied to the new formulation in order to improve its numerical resolubility. Numerical examples that arise from the well known Solomon benchmark collection are presented and discussed in Section 5. Finally, the conclusions are outlined in section 6.

II. PROBLEM DEFINITION AND ITS USUAL MATHEMATICAL MODEL

Consider a route-network represented by an undirected graph G$\{I \cup p, A\}$ with $I = \{i_1, i_2, ..., i_n\}$ denoting the set of nodes or customers and p representing a source /sink node called "the depot". Nodes and the depot are connected by a set of arcs $A = \{(i, j) / i,j \in I \cup p\}$. Known load and price vectors $L = [l_1, l_2, \ldots, l_n]$ and $\Pi = [\pi_1, \pi_2, \ldots \pi_n]$ are associated to the customer set I. Loads l_i must be collected within a time window $[a_i, b_i]$ on each node $i \in I$. The parameters a_i stands for the earliest possible start-time of the service and the parameters b_i states the latest possible start-time of the service at the node. In addition, travel-costs $C = \{c_{ij}\}$ and travel times $\Gamma = \{t_{ij}\}$ are given data for any route segment $(i,j) \in A$. Moreover, the service time on node i is denoted st_i. For