METHODOLOGY BASED ON SVD FOR CONTROL STRUCTURE DESIGN

L.A. ALVAREZ[†§] and J. ESPINOSA[‡]

National University of Colombia. Faculty of Mines. Cra 80 No 65 - 223 Medellín Colombia
† Process and Energy School. luz@pqi.ep.usp.br
‡ Electric and Control Engineering School. National University of Colombia. jjespino@unal.edu.co
§ Present address: LSCP/CESQ Chemical Engineering Department, Polytechnic School. University of São Paulo. Av. Prof. Luciano Gualberto trav 3, 380. CEP 05508-900 São Paulo (SP) Brazil. Tel. +55-11 30912237. Fax. +55-11 38132380.

***Abstract*— This work presents a methodology for control structure design based on the Singular Value Decomposition (SVD) of the Hankel matrix, constructed from the observability and controllability matrices. A weight index is proposed as a measure of the impact of each input and output variable and, based on it, an input-output pairing selection is performed. The methodology is validated using the Tennessee Eastman process and the result is compared to the relative gain method in frequency.**

***Keywords*— Control structure design, SVD, Hankel matrix, Tennessee Eastman process.**

I. INTRODUCTION

The growing demand for optimal operation and effective use of energy and raw materials in chemical processes has generated a need to design processes with tighter integration. Materials recycle as well as energy exchange between the different process streams, modify the dynamic behavior of the process and make it more difficult to control. Until now, the vast majority of control studies have focused in controller design and little attention has been paid to control structures. The relationship between controlled, measured and manipulated variables and how to link these variables to form control loops in plants are problems that, in practice, are solved heuristically, because there is not enough solid theory. The main existing methods for control structure design are based in relative gains (Bristol, 1966) and singular value analysis (Skogestad and Postlethwaite, 1996). Although these methods are well grounded, the process system representation used does not take completely into account the dynamic behavior of the process. So, existing methods could indicate less appropriate control structures.

The objective of this work is to present the development of a methodology for control loop configuration, based on a singular value analysis of the Hankel matrix. In discrete time, that matrix represents the input-output behavior of the process and implicitly contains the dynamics, which makes it a very generic representation. The methodology is validated in the partially controlled Tennessee Eastman process (Larsson *et al.*, 2001) in two levels: external and internal loops. At the external level the objective is to build master control loops with variables with strong interaction, and at the internal level local or slave control loops which are heuristically trivial.

In Section II, the existing tools for control structure design are presented and later, in Section III, the new methodology based on the Hankel matrix is proposed. Section IV describes the system used for methodology validation and in Section V the results of the validation are shown. Finally, Section VI shows the conclusions.

II. METHODS FOR CONTROL STRUCTURE DESIGN

This section discusses some existing methods for the control structure design (CSD).

A. Relative Gains

The method was originally proposed by Bristol (1966) and determines which variables are most convenient for pairing control loops in the process. Relative Gain Array (RGA) gives a measure of the interaction for each possible pairing. The original formulation uses the transfer function matrix of the process, evaluated at steady state, ignoring the dynamic behavior. In order to correct this and other limitations, several extensions have been proposed: RGA at Crossover frequency (Grosdidier *et al.*, 1985), Relative gains for integrating processes (Arkun and Downs, 1990), RGA for non-square plants (Cao, 1995), RGA with operating frequency optimization (McAvoy *et al.*, 2003), RGA integrating gains from zero to the frequency bandwidth (Xiong *et al.,* 2005).

B. Singular Value Decomposition

SVD is a matrix factorization that allows the determination of the singular values of any matrix. Consider the complex matrix $\mathbf{G}$, it can be decomposed as $\mathbf{G} = \mathbf{U}\Sigma\mathbf{V}^{\mathbf{T}}$. The diagonal matrix Σ contains the singular values σ_i ordered from largest to smallest, and two matrices $\mathbf{U}$ y $\mathbf{V}$ that are orthonormal. In practical terms, when a process system is represented by $\mathbf{G}$, it can be said that each singular value σ_i represents a mode i of operating the process and, according to this, the largest singular values indicate the most “energetic” modes. The vectors U_i of $\mathbf{U}$ and V_i of $\mathbf{V}$ represent the direction of each mode i. In that way the V_i indicate the direction of the process inputs and the U_i indicate de direction of the process outputs. This interpretation has generated the different CSD methods that use SVD as a tool. The CSD criteria are based in the maximum singular value, the condition number and the singular vectors:

- *Maximum singular values*: In general, it is desirable that the maximum singular value be small. In Havre *et*