GENERALIZATION OF INTERNAL MODEL CONTROL LOOPS USING FRACTIONAL CALCULUS

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Abstract— **Process control represents an important tool for meeting product quality, process safety and environmental regulation. Different control strategies have been recently proposed in the literature; however, internal model control (IMC) has received great attention. Fractional calculus represents a fast growing field involving non-integer order derivatives. The aim of this work is the application of fractional calculus to develop generalized internal model control loops transfer functions, which is presented in two different approaches: firstly, the process model is considered perfect, i.e., equal to the internal model; secondly, the internal model is described by fractional transfer function. Simulation results showed that the proposed generalization could successfully control an industrial oven and a biochemical reactor described by fractional models, allowing better results when compared to integer order IMC.**

Keywords— **Process control, fractional differential equation, Caputo representation, Internal Model Control, Generalization.**

I. INTRODUCTION

Control systems play a key role in chemical and biochemical plants operation focusing on high production meeting product quality, process safety and environmental regulation, (Lenzi *et al.*, 2005). Literature reports different conventional control algorithms, which have been successfully applied to the control of industrial systems (Seborg, 1999), being internal model control (IMC) originally reported by Garcia and Morari (1982). It must be highlighted that IMC is a model-based design technique and it also allows model uncertainty and trade-offs between performance and robustness to considered in a more systematic fashion (Zheng and Hoo, 2004). Literature also reports extensions of IMC to control of nonlinear systems (Nitsche *et al.*, 2007), tuning rules (Vilanova, 2008), gain scheduled control (Xie and Eisaka, 2008), control of unstable systems (Wang *et al.*, 2001), use of IMC in feedforward control loops (Mawire and McPherson, 2008) and applications with fuzzy logic (Duan *et al.*, 2008), among others.

Fractional differential order equations represent a fast growing research field nowadays (Das, 2008). Different applications of fractional calculus have been reported, i.e., diffusion studies (Lenzi *et al.*, 2006), rheology (Craiem *et al.*, 2008), process identification (Isfer *et al.*, 2010a), process control (Isfer *et al.*, 2010b), electroanalytical chemistry (Oldham and Spanier, 2006), among others (Hilfer, 2000). Further details regarding the formalism of fractional calculus are beyond the scope of this work and can be found elsewhere (Oldham and Spanier, 2006). Fractional control has been successfully applied to mechanical (Pommier *et al.*, 2002) and eletromechanical systems (Sabatier *et al.*, 2004). The use of IMC control and fractional calculus was firstly reported by Valerio and Sa da Costa (2006). In their manuscript, IMC control was only used as an alternative tool for fractional PID controller tuning. To the best of our knowledge, applications of IMC control loops (Brosilow and Joseph, 2002) only use classical calculus, no studies of IMC control loops involving fractional calculus were found in the open literature.

The aim of this work is the application of fractional calculus to develop generalized internal model control loops transfer functions. The study is divided into two parts. In the first, the process model is considered perfect, i.e., equal to the internal model. In the second part, the internal model is described by fractional transfer function. Finally, the proposed generalization is applied to simulation studies in order to control an industrial oven and a biochemical reactor described by fractional models.

II. THEORETICAL FRAMEWORK

A fractional derivate can be obtained by several approaches. However, in this work, only the Caputo representation, presented bellow, will be considered.

$$ {}_aD_x^{\beta} f(x) = \frac{1}{\Gamma(m-\beta)} \cdot \int_a^x \frac{f^{(n)}(\tau)}{(x-\tau)^{\beta+1-m}} d\tau \qquad (1)$$

where $m \le \beta \le m+1$; $\beta \in \Re$; m $\in \aleph$.

The first advantage of this representation is the fact that initial conditions of the fractional differential equations can be expressed in terms of integer order derivatives, which usually have physical interpretation. Secondly, for this representation, the fractional derivate of a constant function is zero allowing the use of the classical deviation variables approach, simplifying the solution of the mathematical problem. The IMC loop considered for this work is presented by Fig. 1.

The transfer functions of the controller and actuator can be grouped into only one term given by their prod-