

# INFLUENCES OF TEMPERATURE DEPENDENT VISCOSITY AND THERMAL CONDUCTIVITY ON THE UNSTEADY FLOW AND HEAT TRANSFER OF A MICROPOLAR FLUID OVER A STRETCHING SHEET

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**Abstract**— This paper is devoted to describe the analysis of unsteady boundary layer flow and heat transfer of a viscous, incompressible micropolar fluid having temperature-dependent viscosity and thermal conductivity over a non-isothermal horizontal stretching sheet. The fluid viscosity and the thermal conductivity are assumed to vary as inverse linear functions of temperature. The unsteady boundary layer equations for the momentum, angular momentum and thermal energy are simplified using suitable transformations. The resulting system of nonlinear ordinary differential equations is solved numerically by employing a shooting technique with fourth order Runge-Kutta integration scheme. The effects of the various parameters on the velocity, microrotation and temperature profiles are presented graphically. In addition, tabulated results for the skin-friction coefficient, Nusselt number and couple stress at the plate are presented and discussed.

**Keywords**— Unsteady flow, micropolar fluid, stretching sheet, temperature dependent viscosity and thermal conductivity.

## I. INTRODUCTION

The theory of microfluids, as developed by Eringen (1964) has been a field of active research for the last few decades as this class of fluids represents mathematically many industrially important fluids, like paints, blood, body fluids, polymers, colloidal fluids and suspension fluids. This theory takes into account the initial characteristics of the substructure particles which are allowed to undergo rotation. Micropolar fluids are fluids with microstructure belonging to a class of fluids with non-symmetrical stress tensor. Physically, they represent fluids consisting of randomly oriented particles suspended in a viscous medium. Gorla (1995) has studied the unsteady mixed convection in micropolar boundary-layer flow over a vertical surface where unsteadiness arises as a result of time-varying nature of the free stream velocity as well as the wall temperature. Kim (2001) considered unsteady convection flow of micropolar fluid past a vertical porous plate embedded in a porous medium. The flow of a magneto-micropolar fluid past a continuously moving plate is analyzed by Seddeek (2003). The study of the mixed convection flow of a continuously moving flat plate placed in a parallel

moving stream of a micropolar fluid was provided by Bhargava *et al.* (2003). The natural convection flow of a micropolar fluid past a continuously moving vertical porous flat plate has been studied numerically by Rahman and Sattar (2006). Ishak *et al.* (2007) have studied the problem of steady boundary-layer flow of a micropolar fluid on a continuously moving or fixed permeable surface. EL-Kabeir and Gorla (2007) studied the effects of transverse magnetic field on natural convection of a micropolar fluid-saturated porous medium around heated vertical surfaces. The steady two-dimensional mixed convection flow of a micropolar fluid over a nonlinear stretching sheet is investigated by Hayat *et al.* (2008). Patil and Kulkarni (2008). The steady mixed convection boundary layer flow over a vertical surface immersed in an incompressible micropolar fluid is considered by Ishak *et al.* (2009).

Most of the existing analytical studies for this problem are based on the constant physical properties of the ambient fluid. However, it is known (see, Herwing and Gersten, 1986) that these properties may change with temperature, especially for fluid viscosity. To accurately predict the flow and heat transfer rates, it is necessary to take into account this variation of viscosity and thermal conductivity. Hossain and Munir (2001) studied the natural convection flow of a viscous fluid about a truncated cone with temperature dependent viscosity and thermal conductivity. The interaction of forced convection and thermal radiation during the flow of a surface moving continuously in a flowing stream of micropolar fluid with variable viscosity have been considered by EL-Kabeir (2004). Elbarbary and Elgazery (2004) studied the effects of variable viscosity and variable thermal conductivity on heat transfer from moving surfaces in a micropolar fluid. Hassanien *et al.* (2003) investigated variable viscosity and thermal conductivity effects on combined heat and mass transfer in mixed convection over a wedge in porous media. The effects of variable viscosity and thermal conductivity effects on MHD flow and heat transfer in viscoelastic fluid over a stretching sheet were investigated by Salem (2007). Rahman and Sattar (2007) studied the transient convective flow of micropolar fluid past a continuously moving vertical porous plate in the presence of radiation Elgazery (2009) analyzed the effects of variable viscosity and thermal diffusivity on MHD flow of a micropolar fluid