AN IMPROVED SCHEME FOR SOLVING ATMOSPHERIC RADIATIVE TRANSFER PROBLEMS WITH THE SPECTRAL NODAL METHOD

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***Abstract* – In this article, we report on recent advances in a spectral nodal method for the numerical solution of multislab atmospheric radiation problems. Here, we derive a set of periodic relations for the coefficients of the bidirectional functions of our method, and we use these periodic relations to improve a recently developed computational scheme for solving a set of multislab atmospheric radiation problems with an arbitrary number and type of optically stationary layers. We present numerical results for a set of prototype problems that show the effect of stratospheric ozone depletion on the amount of ultraviolet-B radiation that reaches the Earth's surface.**

***Keywords*— Multislab atmospheric radiation; Discrete ordinates; Periodic relations; Stationary layers; Ultraviolet-B radiation.**

I. INTRODUCTION

In recent years, we have developed a spectral nodal method for numerically solving discrete ordinates radiative transfer problems in a stratified plane-parallel (multislab) grey planetary atmosphere (de Abreu, 2004a, b). In order to cast the discretized equations of our method in a suitable form for computer implementation and solution of practical problems in an efficient manner, we have successfully derived bidirectional functions for an arbitrary layer of a multislab model atmosphere (de Abreu, 2005a, b, 2006a). These bidirectional functions give us the emerging diffuse intensities at the bottom and top edges of a layer in terms of the incident diffuse intensities at top and bottom, and in terms of corresponding source terms. Moreover, they allow us to: i) replace corresponding atmospheric layers in multislab radiative transfer computations without loss of numerical accuracy; and ii) make an efficient use of our spectral nodal method for solving multislab atmospheric radiation problems with an arbitrary number and type (internal and/or boundary) of optically stationary layers, i.e. those layers in a multislab model atmosphere whose optical properties do not change from one problem to another in the set (de Abreu, 2006b).

In this article, we report on recent advances in our spectral nodal method. In Section II, we describe the class of radiative transfer problems that we are concerned with, and we outline our spectral nodal method. In Section III, we derive in full a set of periodic relations for the coefficients of the bidirectional functions of our spectral nodal method. In Section IV, we give a detailed account of an improved scheme for solving a set of multislab atmospheric radiation problems with optically stationary layers. In Section V, we present numerical results for a set of prototype problems in the ultraviolet-B (UV-B) range, and in Section VI we conclude this article with a discussion.

II. PROBLEM FORMULATION AND SPECTRAL NODAL METHOD

A. Problem Formulation

Let us assume that the multislab model atmosphere referred to at the onset consists of R contiguous, disjoint, and uniform layers so that layer 1 is the uppermost layer and layer R is the lowermost one. We assume further that the diffuse component of the intensity of the radiation field in the multislab domain can be accurately described by the discrete ordinates radiative heat transfer equations (Chandrasekhar, 1950; Siewert, 2000; Stamnes *et al.*, 1988; Thomas and Stamnes, 1999)

$$\mu_m \frac{d}{d\tau} I^d_{r,m}(\tau) + I^d_{r,m}(\tau) = \tag{1}$$
$$\frac{\varpi_r}{2}\sum_{\ell=0}^{L_r}(2\ell+1)\beta_{\ell,r}P_\ell(\mu_m)\sum_{n=1}^{N}\omega_n P_\ell(\mu_n) I^d_{r,n}(\tau)$$
$$+F_0\frac{\varpi_r}{2}\sum_{\ell=0}^{L_r}(2\ell+1)\beta_{\ell,r}P_\ell(\mu_0)P_\ell(\mu_m)*$$
$$\exp\left[-\frac{1}{\mu_0}(\tau-\tau_0)\right], m=1:N,\ \tau_{r-1}\le\tau\le\tau_r, r=1:R.$$

We should note that the direct component of the radiation field is rather straightforward to model and compute (Thomas and Stamnes, 1999). In the set of Eqs. (1), N is the order of the angular quadrature, m from 1 to $N/2$ holds for the downward directions of radiation propagation, and m from $N/2$ +1 to N holds for the upward ones; τ is the optical variable, and τ_r–1 and τ_r denote, respectively, the optical depths of the upper and lower boundaries of the rth layer; the symbol μ_m indicates the cosine of the angle defined by the discrete direction m and the downwardly oriented τ axis, and the quantity ω_n is the angular weight for direction n.

In this article, we use Gauss-Legendre quadrature sets (Lewis and Miller Jr., 1993), and we have ordered the discrete directions so that $\mu_{m-1} < \mu_m$, m from 2 to $N/2$, and $\mu_{m+N/2} = -\mu_m$, m from 1 to $N/2$. Accordingly, $\omega_{n+N/2} = \omega_n$, n from 1 to $N/2$, for the angular weights; ϖ_r