

# HEAT AND MASS TRANSFER IN MHD FREE CONVECTION ALONG A VERTICAL WAVY PLATE WITH VARIABLE SURFACE HEAT AND MASS FLUX

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**Abstract**— The problem of combined heat and mass transfer in buoyancy-induced MHD natural convection flow of an electrically conducting fluid along a vertical wavy plate with power-law variation of both heat and mass flux was investigated. The resulting transformed governing equations are solved numerically by an implicit finite-difference scheme. The results are presented for the major parameters including the wave amplitude  $a$ , the magnetic parameter  $Mn$ , the buoyancy ratio between species and thermal diffusion  $Br$ , the Lewis number  $Le$ , and the power-law parameter  $\lambda$ . A systematic study on the effects of the various parameters on flow, heat and mass transfer characteristics is carried out.

**Keywords**— Wavy plate, Heat mass flux, Magnetic field, porous media.

## I. INTRODUCTION

Investigation of magneto-hydrodynamic flow for an electrically conducting fluid past a heated surface has attracted the interest of many researchers in view of its important applications in many engineering problems such as plasma studies, petroleum industries, MHD power generators, cooling of nuclear reactors, the boundary layer control in aerodynamics, and crystal growth. This study has been largely concerned with the flow and heat transfer characteristics in various physical situations. For example, Watanabe and Pop (1994) investigated the heat transfer in thermal boundary layers of magneto-hydrodynamic flow over a flat plate. Kumari *et al.* (1990) treated convection in porous media near a horizontal uniform heat flux surface. Elbashbeshy (1997) studied heat and mass transfer along a vertical plate in the presence of a magnetic field. Chamkha (1997) considered the problem of MHD natural convection from an isothermal inclined plate embedded in a thermally stratified porous medium. Chamkha and Khaled (2001) investigated the problem of coupled heat and mass transfer by hydromagnetic free convection from an inclined plate in the presence of internal heat generation or absorption, and similarity solutions were presented. Chen (1998) studied the mixed convection heat transfer from a horizontal plate with variable surface heat flux in a porous medium. However, the monograph by Gebhart *et al.* (1998) and Nield and Bejan (1999) provided an overview of the early studies con-

cerning the natural convection boundary-layer flow due to simultaneous heat and mass transfer over heated surfaces with various geometries.

In the other hand, few studies have been carried out to examine the effect of geometric complexity, such as irregular surfaces, on the convection heat transfer. That is because complicated boundary conditions or external flow fields are difficult to deal with. However, the prediction of heat transfer from an irregular surface is of fundamental importance, and is encountered in several heat transfer devices, such as flat-plate solar collectors and flat-plate condensers in refrigerators. Moreover, surfaces are sometimes intentionally roughened to enhance heat transfer for the presence of rough surfaces disturbs the flow and alters the heat transfer rate. However, all of the previous studies considered only the case of a flat plate or simple two-dimensional bodies, and few have been done on wavy surfaces.

Cheng (2000) studied the phenomenon of natural convection heat and mass transfer near a vertical wavy surface with constant wall temperature and concentration in a porous medium. Rees and Pop (1994a) investigated the effects of transverse surface waves on the free convective boundary layer induced by a uniform heat flux vertical surface embedded in a porous medium. Rees and Pop (1994 b,c and 1995) done many of related papers. The magnetohydrodynamic (MHD) flow and heat transfer from a horizontal wavy surface with variable heat flux due to the effect of magnetic fields on flow control and performance is considered by Bourhan and Al-Odat(2004). Wang and Chen (2005) reported the effect of magnetic field on forced heat and fluid flow over a wavy surface subjected to a heat flux proportional to  $(1 + x^2)^n$  (where  $n$  is constant). Hossain and Pop (1996) and Hossain *et al.* (1997) investigated the problem of magnetohydrodynamic boundary layer flow and heat transfer on a continuous moving wavy surface and the problem of magnetohydrodynamic free convection along a vertical wavy surface.

In the formulation of our problem it can be demonstrated that the system of momentum, heat and mass conservation equations can be reduced to a five parameter problem by introducing suitable transformation variables. The major problem parameters include the amplitude of the wavy surface  $a$ , the magnetic parameter  $Mn$ , the Lewis number  $Le$ , the buoyancy ratio  $Br$ , the heat and mass flux exponent  $\lambda$ . The resulting transformed