A SOLUTION FOR A HEAT TRANSFER MODEL IN A MOVING BED THROUGH THE SELF-ADJOINT OPERATOR METHOD.*

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***Abstract*— Usually, heat and/or mass transfer models with time dependence, in a fixed, moving or cross-flow beds, are solved analytically by the use of the Laplace transform method. When the determination of the character of the poles is not on easy problem, this method presents the transform inversion using the residue theorem as the major application difficulty. In this work, an alternative method is discussed which casts the system of equations into a matrix problem of the Sturm-Liouville type. As an example, the solution of a heat transfer model in a moving bed is presented. The advantage this approach is a direct solution of the temperature profiles in the particle and in the bulk fluid near the solid-fluid interface by using a spectral expansion in terms of the self-adjoint matrix operator involved, with guaranteed convergency, and it can be used easily as an interpolation scheme to solve numerically advection/diffusion problems.**

***Keywords*— Heat transfer, Moving bed, Multiphase reactors, Analytical solution, Self-adjoint.**

I. GENERAL CONSIDERATIONS

As for the heat and/or mass transfer between a particulated solid phase and one or more fluid phases (gaseous or liquid), a great variety of technological applications is observed, mainly due to the high efficiency of those transfer processes. We can, for example, emphasize the pyrolysis of oil shale fine particles in a moving bed reactor (Bertoli, 2000), the Fischer Tropsch synthesis in well-mixed slurry reactors (Ahn *et al.*, 2005), the cross-flow continuous fluidized bed dryer (Izadifar and Mowla, 2003), the catalytic cracking of petroleum in a circulating fluidized bed reactor (Michalopoulos *et al.*, 2001; Becerril *et al.*, 2004), and others.

The mathematical modeling of the above exemplified transfer processes follows basically models of two or more phases that are related by interphase transfer processes; in these models, the spatial variation of the variables are considered in the distributed parameter model or neglected by the lumped parameter model. Real processes are distributed, but some of them can be approximated as lumped ones (in one or more phases). A situation in which the temperature gradient in the solid phase may be neglected will be commented in the following.

Bertoli (1989, 2000) presents a comparison between a lumped parameter model and a distributed parameter one, for the heat transfer process in a moving bed, based on experimental data obtained by Lisbôa (1987). In that analysis, the analytical solutions were obtained by the Laplace transform method with inversion by the residue theorem, and the results for both models proved a good concordance between the experimental data and those predicted by the model. We intend to demonstrate the application of an alternative method for the solution of partial differential equation systems from self-adjoint operators, that permits a maximum preservation of the physical characteristics of the problem without, however, needing excessive numerical calculations. This method was mathematically detailed by Arce and Ramkrishna (1986, 1988) for the heat transfer in a fixed bed reactor.

The motivation to apply analytical methods for this kind of advection/diffusion problem resumes, in fact, in two aspects:

- the first one in a mathematical point of view, the analytical solution can be easily extended for numerical methods as interpolation scheme in the CFD techniques to increase convergence rate and stability of the numerical solutions;
- in a physical point of view, this heat transfer model represents the interface between gas-solid phases and it can be applied for several physical situations.

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