LINEARLY IMPLICIT DISCRETE EVENT METHODS FOR STIFF ODE'S.

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Abstract— **This paper introduces two new numerical methods for integration of stiff ordinary differential equations. Following the idea of quantization based integration, i.e., replacing the time discretization by state quantization, the new methods perform first and second order backward approximations allowing to simulate stiff systems. It is shown that the new algorithms satisfy the same theoretical properties of previous quantization–based integration methods. The translation of the new algorithms into a discrete event (DEVS) specification and its implementation in a DEVS simulation tool is discussed. The efficiency of the methods is illustrated comparing the simulation of two examples with the classic methods implemented by Matlab/Simulink.**

Keywords— **Stiff System Simulation, Quantization Based Integration, DEVS**

I. INTRODUCTION

The use of traditional methods (Hairer *et al.*, 1993; Hairer and Wanner, 1991; Cellier and Kofman, 2006) based on time discretization to integrate stiff systems require the use of implicit algorithms since the required step size used by explicit methods is limited by the stability region and the resulting step size becomes inadmissibly small (Cellier and Kofman, 2006).

In fact, numerical integration methods that include in their numerically stable region the entire left half $(\lambda \cdot h)$ plane (or at least a large portion of it) are necessary for stiff systems integration (Cellier and Kofman, 2006). Only some implicit methods have this type of stability region. Explicit algorithms showing that feature do not exist.

The problem with implicit methods is that they are computationally expensive because in each step they need to use iterative algorithms to determine the next value (usually with the Newton Iteration). The problem becomes critical in applications related to real time simulation, where in many cases performing iterations becomes unacceptable.

An alternative approach to classic time slicing started to develop since the end of the 90's, where time discretization is replaced by state variables quantization. As a result, the simulation models are not discrete time but discrete event systems. The origin of this idea can be found in the definition of Quantized Systems (Zeigler *et al.*, 2000).

This idea was then reformulated with the addition of hysteresis –to avoid the appearance of infinitely fast oscillations– and formalized as the Quantized State Systems (QSS) method for ODE integration in (Kofman and Junco, 2001). This was followed by the definition of the second order QSS2 method (Kofman, 2002), the third order QSS3 method (Kofman, 2006).

The QSS methods showed some important advantages with respect to classic discrete time methods in the integration of discontinuous ODEs (Kofman, 2004), sparsity exploitation (Kofman, 2002), the property of absolute stability, and the existence of a global error bound (Cellier and Kofman, 2006).

In spite of these properties, QSS, QSS2 and QSS3 fail when applied to stiff systems due to the appearance of fast oscillations. They remain numerically stable at the expense of utilizing excruciatingly small step sizes, as any explicit algorithm is expected to require in this situation.

To solve this problem, a first order backward QSS method (called BQSS, after Backward QSS) was proposed in Migoni *et al.* (2007). The method was able to efficiently integrate many stiff systems. The basic idea of the BQSS method is to use a *future* value of the states to obtain the quantized value. Yet, whereas the algorithm is implicit, it still can be implemented without Newton iterations. The reason is that each state has only two possible future values. If a state variable currently assumes a value of q_j, the next value of that state variable must be either $q_j + \Delta q_j$, or $q_j - \Delta q_j$. Hence all possible next values can be *enumerated* without an open-ended search. In other words BQSS was the first explicit method for stiff ODEs.

The main drawback of BQSS is that it performs only a first order approximation and accurate results cannot be obtained. Another problem is that BQSS introduced an extra perturbation term that increases the error bound and, in some nonlinear systems, might provoke the appearance of spurious equilibrium points.

This paper presents first a new method that combines the idea of BQSS and linearly implicit integration.

The first order accurate linearly implicit QSS