NUMERICAL INVESTIGATION OF INERTIA AND SHEAR-THINNING EFFECTS IN AXISYMMETRIC FLOWS OF CARREAU FLUIDS BY A GALERKIN LEAST-SQUARES METHOD

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Abstract— **This article presents a finite element simulation of Carreau flows through an abrupt contraction. The employed mechanical model consists in using the Carreau viscosity equation to characterize the shear-thinning fluid behavior, giving rise to a generalization of Navier-Stokes equation containing a non-linear diffusion term. A Galerkin-Least Squares methodology approximates the mechanical model circumventing the Babuška-Brezzi condition, which consists of adding to the classical Galerkin method mesh-dependent residuals, resulting from least squares of the Euler-Lagrange equations. Numerical results for both velocity and pressure fields accounting for shear-thinning and fluid inertia effects have been obtained for an axisymmetric 4:1 sudden contraction with Carreau number ranging from 0 to 100, power-law exponent from 0.2 to 1.0 and Reynolds number from 2 to 100. These results have shown good agreement with the literature.**

Keywords — **Non-Newtonian fluids, Carreau equation, sudden contraction flow, Galerkin-Least Squares method.**

I. INTRODUCTION

Non-Newtonian fluid behavior is present in a wide class of fluids, encompassing suspensions of particles (e.g. blood, paint, ink, and food products), polymer melts and solutions. Many mathematical models describe the non-linear behavior of the material functions obtained experimentally, some of them built within the framework of an axiomatic approach of the non-linear theory of continuous media.

Numerical approximations for shear-thinning flows have deserved a special attention of a large number of researches in non-Newtonian fluid mechanics. Kim *et al.* (1983) have employed a classical finite element approach to study the roles of fluid inertia and shear-rate dependence of a Carreau viscosity field. The authors have concluded that the effect of increasing either shear-thinning or fluid inertia decreases the upstream vortex size at the sudden contraction flow. Laminar fully-developed flows in an eccentric annular geometry for power-law fluid have been considered by Fang *et al.* (1999). The employed finite difference scheme gives rise to numerical results for a range of annular radius ($0.2 \leq r \leq 0.8$), eccentricity ($0 \leq \varepsilon \leq 0.8$) and power-law coefficient ($1 \geq n \geq 0.2$), with these two latter parameters playing a strong influence on the fluid behavior. Roberts *et al.* (2001) introduce pseudoplastic empiric models based on Cross and Ellis viscosity functions, taking into account the material yield stresses, used to describe commercial fluids such as drilling mud. Reis Junior and Naccache (2003) simulated, via a finite volume methodology, non-Newtonian fluids through axisymmetric sudden expansion and contraction flows, investigating the influence of rheological parameters on these flows. Power-law fluid flows through a 3:1 sudden expansion were simulated by Manica and De Bortoli (2004), employing a finite difference technique coupled to a Runge-Kutta scheme. The performed simulations have shown bifurcation for values of power-law index between 0 and 2, which occurs either at the beginning of shear thinning or for strong shear thickening. A finite volume method, employing the SIMPLE pressure-correction strategy coupled to the QUICK difference scheme, has been used by Neofytou (2005) to study four non-Newtonian viscous models in a lid-driven cavity flow: power-law, Quemada and modified Bingham and Casson ones.

In the present work the roles of fluid inertia and shear-rate dependency on viscosity for an inelastic non-Newtonian fluid have been analyzed. A Galerkin/least-squares (GLS) methodology was employed to simulate pseudoplastic flows obeying Carreau viscosity function through an axisymmetric 4:1 sudden contraction. This GLS methodology overcomes classical Galerkin shortcomings at high advective flows by adding mesh-dependent terms (functions of the residuals of the Euler-Lagrange equations evaluated elementwise) to classical Galerkin formulation, enhancing its convergence without upsetting its consistency, since the residuals of the Euler-Lagrange equations are satisfied by their exact solutions. Stabilization is important not only to circumvent Babuška-Brezzi condition but also to preserve numerical stability in locally advective dominated flow regions due to the material behavior of shear-thinning liquids.