PETROV-GALERKIN APPROXIMATION FOR ADVECTIVE-DIFFUSIVE HEAT TRANSFER IN SATURATED POROUS MEDIA

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Abstract— **This article studies the heat transport in a flow through a saturated rigid porous medium. The mechanical model is based on the Continuum Theory of Mixtures which considers the fluid and the porous matrix as overlapping continuous constituents of a binary mixture. A Petrov-Galerkin formulation is employed to approximate the resulting system of partial differential equations, overcoming the classical Galerkin method limitation in dealing with advective-dominated flows. The employed method is built in order to remain stable and accurate even for very high advective-dominated flows. Taking advantage of an appropriated upwind strategy, the applied finite element method proved to generate accurate approximations even for very high Péclet regime. Some two-dimensional simulations of the advective-diffusive heat transfer in a flow through a porous flat channel employing lagrangean bilinear and serendipity biquadratic elements have been performed attesting the reliability of the employed Petrov-Galerkin formulation as well as the poor performance of Galerkin one even when mesh refining is considered.**

Keywords— **Porous media, mixture theory, computational heat transfer, finite elements, Petrov-Galerkin formulation.**

I. INTRODUCTION

Transport phenomena in porous media play an important role in many a field of engineering science, such as geomechanics, petroleum and mining industries, sintering technologies and biomechanics. Besides, interactions among fluids and solids are present in many industrial processes and the fluids may be passed over packed beds of solid material in order to improve processes like heat and mass transfer or chemical reactions. Nowadays an increasing attention is being devoted to transport in porous media motivated by the importance of problems that impact the energy self-sufficiency and the environmental state. Some practical applications like packed-bed heat exchangers, enhanced oil recovery processes, storage of nuclear waste material, contamination of soils by hazardous wastes and pollution movement stimulate the interest attached to these phenomena.

Most of the works dealing with transport in porous media describe quantities such as temperature, pressure, concentration and the velocity components as volumetric averages (Whitaker, 1969); in order to describe the phenomena by employing a classical continuum mechanics approach. These models substitute the balance of linear momentum by Darcy's law with the addition of empirically determined terms – Brinkmann and Forchheimer extensions – to account for inertia and viscous effects and to satisfy the no-slip condition (Vafai and Tien, 1981). Nield (2000) analyzed viscous dissipation and nonlinear drag for Darcy, Brinkmann and Forchheimer models for incompressible fluid flows through porous media. The so-called volume averaging technique has already allowed the analysis of complex problems. Examples are the multiphase transport process with phase change in unsaturated porous media (Vafai and Whitaker, 1986), the forced convection considering heat sources and a partially porous channel (Hadim, 1994) or axial and radial dispersion (Adani *et al.*, 1995), the mixed convection (Aldouss *et al.*, 1996; Chang and Chang, 1996; Chen *et al.*, 1996) as well as variable porosity effects (Vafai, 1984). Thermally developing forced convection in a porous medium was studied by Nield *et al.* (2003) employing a modified Graetz method with Brinkmann model, for parallel plate channel and circular tube, both with walls at constant heat flux. This work was subsequently extended considering walls at constant temperature (Nield *et al.*, 2004a), and both boundary conditions were considered by Nield *et al.* (2004b). The entropy generation – considering viscous dissipation effects (Brinkmann extension) – was analyzed by Mahmud and Fraser (2005) who obtained analytical expressions for velocity, temperature and Nusselt number. Hooman and Ejlali (2007) analyzed thermally developing forced convection in a porous matrix employing both First and Second Laws of Thermodynamics and including Brinkmann (viscous dissipation) effects, using the perturbation solution of Hooman and Ranjbar-Kani (2004), in order to compute