Latin American Applied Research 38:141-145 (2008)

FRACTIONAL CALCULUS APPLIED TO MODEL ARTERIAL VISCOELASTICITY

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Abstract— **Arterial viscoelasticity can be described using stress-relaxation experiments. To fit these curves, models with springs and dashpots, based on differential equations, were widely studied. However, uniaxial tests in arteries show particular shapes with an initial steep decay and a slow asymptotic relaxation. Recently, fractional order derivatives were used to conceive a new component called *spring-pot* that interpolates between pure elastic and viscous behaviors. In this work we modified a standard linear solid model replacing a dashpot with a *spring-pot* of order α. We tested the fractional model in human arterial segments. Results showed an accurate relaxation response during 1-hour with least squares errors below 1%. Fractional orders α were 0.2-0.4, justifying the extra parameter. Moreover, the adapted parameters allowed us to predict frequency responses that were similar to reported Complex Elastic Moduli in arteries. Our results indicate that fractional models should be considered as real alternatives to model arterial viscoelasticity.**

Keywords — **Viscoelasticy, Stress-relaxation, Human arteries, Standard-linear solid, Fractional calculus.**

I. INTRODUCTION

Arteries, like other soft tissues exhibit viscoelastic behavior. In this context, the mechanical energy transferred to them is partly stored in a reversible form (elasticity) while other fraction is dissipated (viscosity). Getting insight into viscoelastic properties of arteries can help to identify their biomechanical structure and function, to study the progression and reversion pathologies that might affect them and even to predict their natural deterioration with age and the influence of cardiovascular circulation (Armentano *et al*., 2006; Fung, 1981).

Uniaxial stress-relaxation test can be used to study arterial wall mechanics. Arterial segments are stretched with a loading ramp that stops while true stress is registered. Measured stress in arteries describes a particular curve with a fast steep decrease and a very slow asymptotic relaxation (Hardung, 1952; Jager, 2005; Bergel, 1961). This temporal response to a step deformation in arterial segments can also be associated to their frequency response using complex elastic modulus E^*. In frequency domain, E^* exhibit a fast initial increase from static values, progressing to attain a plateau at higher frequencies (Westerhof and Noordergraaf, 1970). In that sense, arteries are relatively insensitive to strain rate in a wide frequency range.

Models based on ordinary differential equations were used to describe stress-relaxation experiments. They use mechanical analogies connecting springs and dashpots to ultimately represent material viscoelastic properties. The parameters of these components are adjusted using least-squares algorithms and they are eventually associated to some structural or functional properties of the described material.

The simplest model that predicts creep and stress-relaxation is the standard linear solid (SLS) with a parallel combination of a Maxwell arrangement (spring and dashpot in series) with a spring (Fung, 1981). Its temporal step response predicts a negative exponential function. Although this model showed several limitations, it was widely used as a conceptual unit to construct more complicated arrangements that better describe dynamic responses of several materials. Evidently, increasing the number of units (order of the model) blurs the conceptual meaning of each component.

Recently, some models based on fractional order differential equations were presented to describe cell and tissue biomechanics (Djordjevic *et al*., 2003; Koeller, 1984; Suki *et al*., 1994). These equations derive into fractional viscoelastic concepts. Briefly, if a spring represents a zero order element and a dashpot a first order element, a new component called *spring-pot* can be conceived with an intermediate order $1>\alpha>0$. Using α, the mechanical response can interpolate between pure elastic and viscous behaviors. Both temporal relaxation and frequency responses of a *spring-pot* follow power-law functions that seem to be naturally adapted to fit arterial requirements.

The aim of this work was to modify an SLS model, replacing a dashpot with a *spring-pot* of order $1>\alpha>0$ defined using fractional derivatives, to describe arterial viscoelasticity in-vitro. Uniaxial stress-relaxation was registered during 1-hour in human arteries at 2 stress levels and the parameters of the proposed model were adjusted. Finally, an estimation of the frequency response in arteries was presented and discussed.

II. METHODS

A. Modeling

Springs, which represent the elastic component of a viscoelastic material, obey Hooke's Law: