# THE ESTIMATION OF OIL WATER DISPLACEMENT FUNCTIONS

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*Abstract*— **We introduce an algorithm to solve an inverse problem for a non-linear hyperbolic partial differential equation. It can be used to estimate the oil-fractional flow function from the Buckley-Leverett equation. The direct model is non-linear: the sought for parameter is a function of the solution of the equation. Traditionally, the estimation of functions requires the election of a fitting parametric model. The algorithm that we develop does not require a predetermined parameter model. Therefore, the estimation problem is carried out over a set of parameters which are functions. The parameter is inferred from measurements of saturation at different spatial points as a function of time. The estimation procedure is carried out linearizing the solution of the direct model with respect to the parameter and then computing the least-squares solution in functional spaces. The sensitivity equations are derived. We test the algorithm with several numerical experiments.**

*Keywords*— **Parameter estimation, Non-linear equation, Conservation law, Two-phase flow.**

## I. INTRODUCTION

The mathematical simulation of fluid flow through porous media is of vital importance to the management of underground resources, such as aquifers and petroleum reservoirs.

The system of equations that represents the incompressible oil-water flow is derived combining the mass conservation equation with Darcy's law for each phase (Aziz and Settari, 1989). The unknowns are oil and water pressures and saturations. In the equations some parameters, e.g. absolute and relative permeabilities, appear as coefficients. The values of those parameters are essential to perform field-scale analysis.

Fluid saturations are the solution of the differential problem. As the oil-water relative permeability curves are functions of the saturations, the direct model is a non-linear partial differential system. Our goal is to estimate the relative permeabilities from data measured during a displacement test of oil by water in a rock sample.

Historically the model of the physical phenomena was frequently linear with the inverse problem being non-linear; recent work also includes nonlinear physical phenomena models. The problem that we treat here belongs to the last category.

The traditional approach to estimate a function (e.g. relative permeability) requires a parametric model depending on constant values. These values are determined minimizing in finite dimensional spaces (Chardaire-Riviere *et al.*, 1992; Savioli and Bidner, 1994, 1995). This approach has the drawback of imposing a subjective parametric model. Some authors have proposed alternatives to deal with the estimation problem without imposing a parametric model, by using discretized direct models (Kruger *et al.*, 2003; Valestrand *et al.*, 2003).

This work proposes a new alternative to the traditional approach, which is novel in the field of non-linear equations. Up to now, it has been successfully applied to problems where the direct model is based on an initial boundary value partial differential model which is linear (Fernández-Berdaguer, 1998; Fernández-Berdaguer *et al.*, 1995, 1996; Tarantola, 1987).

In our problem there are two main challenges: the dependency of the parameter on the solution and the discontinuity of the initial data. The non-linearity of the equation, that is, the dependency of the parameters on the solution, will present particular difficulties when deriving the sensitivity equations. In addition, we will have to deal with discontinuous initial conditions in the direct model, which may cause discontinuous solutions. Each of these problems will be dealt with separately, by means of assuming simplifying hypotheses for each situation. In this work we will discuss the issue of the parameter depending on the solution.

Under the simplifying hypotheses, the system of equations that represents the oil-water flow can be reduced to a hyperbolic equation in terms of oil saturation and an elliptic pressure equation.

We will deal with the saturation equation which is the one that presents greater difficulty. In this equation, the parameter function is the oil fractional flow. The oil fractional flow is considered a function of the oil saturation only, because it is related to the ratio between water and oil relative permeabilities. That results in the equation being non-linear.

In this work we use the simplifying hypotheses and therefore we proceed with the estimation of the parameter in the non-linear saturation hyperbolic equation of first order.

The forward model corresponds to a laboratory experiment carried on a rock sample.