THE EEG FORWARD PROBLEM: THEORETICAL AND NUMERICAL ASPECTS

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***Abstract*— We focus on the Forward Problem of electroencephalography, discuss a mathematical model and state properties of its weak solutions. A static and a time-dependent model for the source are considered. Numerical solutions, obtained by a Boundary Element Method technique, are compared with the analytical ones and with EEG recordings.**

***Keywords*— EEG, Forward Problem, Epilepsy, Modeling**

I. INTRODUCTION

The electrical activity inside the brain consists of currents generated by biochemical sources at cellular level. This activity can be measured by an electroencephalograph. In the case of epilepsy there are small zones inside the brain that give major contribution in the generation of the electric field. Neurologists have been interested in determining the location of the epileptogenic zones from the measured potential on the scalp in order to avoid invasive techniques.

The problem of determining the source location from the EEG waves is known as the Inverse Problem of Electroencephalography (EEG). In order to solve the inverse problem, we need to have an appropiate model of the Forward Problem (FP) of EEG that consists of calculating the EEG signal when we know the source location.

A typical mathematical model that describes this process is a PDE-boundary value problem of second order, based on the static approximation of Maxwell's Equations (see Hamalainen *et al.*, 1993).

In this work we consider a simplified model of the human head (spherical). We calculate approximated solutions and compare them with the analytical one that exists in the spherical case.

Regarding the source, dipole source models as well as spatially distributed models can be found in Schimpf *et al.* (2002), Lagerlund (1999), de Munck (2002), Yetik *et al.* (2005a) and Yetik *et al.* (2005b). One dimensional source distribution to model the primary cortical response to nerve stimulation is theoretically analyzed in Nolte and Curio (2000). A spatiotemporal source analysis based on the spatiotemporal noise covariance matrix is developed in Huizenga *et al.* (2002). We also propose a time-dependent model for the electric source that approximates a dipole and compare simulated results with real data obtained from EEG recordings provided by Centro Municipal de Epilepsia, Hospital Ramos Meja, Buenos Aires, ARGENTINA.

The paper is organized as follows. In section II the PDE mathematical model is presented. Weak solutions of the FP and their properties are introduced in section III. Section IV includes derivation of the integral equations and its discretization. Different models for the electric source are also described. In section V we present numerical simulations. The approximated solutions for the static dipole model are compared with the analytical ones. The simulated results in the case of a time-dependent source model are compared with real data from EEG recordings. EEG signals plots and 3D plots illustrate the results.

II. THE MATHEMATICAL MODEL

The electric sources inside the brain produce electric and magnetic fields that can be modeled by the Maxwell Equations (see Hamalainen *et al.* , 1993).

The electric current is assumed to be of the form

$$J(x) = \sigma(x)E(x) + J_i(x), \qquad (1)$$

where σ is the conductivity function, σE is the macroscopic electric field and J_i is the impressed current (microscopic level). During an epileptic seizure, *spikes* can be observed along the EEG signals. They are mainly produced by the impressed current.

Due to the high speed of propagation of the electric waves inside the head, there is no delay in the data captured by the EEG recorder. Hence, in order to find the location of the impressed current, we consider the EEG data at the instant at which one of the spikes achieves its highest amplitude. Therefore, a time-independent Maxwell equation may be used to model the relationship between the electric potential u and the impressed current J_i (see Hamalainen *et al.*, 1993), that is,

$$\nabla \cdot (\sigma(x)\nabla u(x)) \;=\; \nabla \cdot J_i(x) \qquad x \in G \quad (2)$$