AN EXTENDED MPC CONVERGENCE CONDITION

A. H. GONZÁLEZ† and J. L. MARCHETTI‡

† *Instituto de Desarrollo Tecnológico para la Industria Química, INTEC (UNL - CONICET)*
alejgon@ceride.gov.ar
‡ *jlmarch@ceride.gov.ar*

***Abstract*— Nominal convergence of Constrained Model Predictive Control has been extensively analyzed in the last fifteen years. The inclusion of a terminal constraint into the optimization problem and the expansion of the prediction horizon up to infinity are the main strategies already proposed in order to achieve the desired stability. However, when a model is used in which the inputs are in the incremental form, these strategies tend to be infeasible. This paper extends the contracting constraint idea by including a simple-to-apply and less restrictive new set of constraints into the optimization problem, to allow nominal convergence.**

***Keywords*— Predictive Control, State Space Model, Receding Horizon, Pseudo Cost Function, Infinite Constraint.**

I. INTRODUCTION

The Receding Horizon idea uses an on-line optimization that updates the manipulated variable at each sample time. In the tracking problem, the difference between the predicted future outputs and the set point is the cost function of a minimization, and nominal stability reduces itself to ensure the convergence of the successive optimal costs. Since consecutive optimization problems are in essence different, it is not simple to compare two consecutive cost functions (the prediction horizon recedes in time so the successive cost functions differ from each other in their location). When an infinite horizon (IHMPC) is used, the end of the consecutive horizons does not vary while the beginning increases. Making use of the Bellman's principle of optimality, which states that the tail of any optimal trajectory is itself the optimal trajectory from its started point, the convergence can be guaranteed. See Maciejowski (2000), pp 191-198. However, when an incremental form of a model is used, the effect of the integrating modes at the end of the control horizon must be zeroed in order to make the infinite open-loop cost bounded (Rodrigues and Odloak, 2003). Similar to the case of terminal constraints, this problem tends to be infeasible and slack variables must be added (Rodriguez and Odloak 2003, Odloak 2004).

Following the strategy developed by González, *et al*. (2004), this paper extends the idea of including a set of contracting constraints to achieve output convergence. In the mentioned work, a preliminary study of convergence conditions that is different from the classical method has been made. However, convergence could not be properly proved. Now, two improvements are made: the convergence of the method is proved, and the whole formulation is translated into a State Space Model in order to take advantage of its well-known benefits.

II. BASIC FORMULATION OF MPC

Consider a system with *nu* inputs and *ny* outputs and consider an optimization cost function as the sum of the future errors inside the prediction horizon plus a manipulated variable penalization, namely

$$J_k = \sum_{i=1}^{p} e\left(k+i/k\right)^T Qe\left(k+i/k\right) + \Delta u_k^T \hat{R} \Delta u_k \qquad (1)$$

where

$$e\left(k+i/k\right) = y\left(k+i/k\right) - r$$
$$\Delta u_k = \left[\Delta u\left(k\right), \Delta u\left(k+1\right), \cdots, \Delta u\left(k+m-1\right)\right]^T \qquad (2)$$

and $y(k+i/k) \in \Re^{ny}$ is the predicted output, $\Delta u(k+i) \in \Re^{nu}$ are the manipulated variable increments, p is the prediction horizon, m is the control horizon, $Q \in \Re^{ny \times ny}$ and $\hat{R} = diag\left(R \cdots R\right)$, $R \in \Re^{nu \times nu}$, are positive definite weighting matrices, and r is the setpoint value.

If Δu^* is the optimal input increment vector, then

$$J_k^* = \sum_{i=1}^{p} e^*\left(k+i/k\right)^T Qe^*\left(k+i/k\right) + \Delta u_k^{*T} \hat{R} \Delta u_k^* \qquad (3)$$

is the optimal cost function value at time k. In the same way, the optimal cost function value at time $k+1$ will be

$$J_{k+1}^* = \sum_{i=2}^{p+1} e^*\left(k+i/k+1\right)^T Qe^*\left(k+i/k+1\right) + \Delta u_{k+1}^{*T} \hat{R} \Delta u_{k+1}^* \qquad (4)$$

Now, following the idea used by Rawling and Muske (1993), an auxiliary pseudo cost function at time $k+1$ is defined using the optimal values of the input changes calculated at time k:

$$\tilde{J}_{k+1} = \sum_{i=2}^{p+1} \tilde{e}\left(k+i/k+1\right)^T Q\tilde{e}\left(k+i/k+1\right) + \Delta\tilde{u}_{k+1}^T \hat{R} \Delta\tilde{u}_{k+1} \qquad (5)$$

where $\tilde{e}\left(k+i/k+1\right)$ is the error at time $k+i$, calculated at time $k+1$, using $\Delta\tilde{u}_{k+1}$, and[1]

[1] The form of the pseudo cost obeys to the fact that, in the infinite horizon case, if no new control increments is introduced at *k+1*, then the optimization remains exactly the same from time *k* to time *k+1*, except for the starting point.