ON-LINE COSTATE INTEGRATION FOR NONLINEAR CONTROL

V. COSTANZA† and C. E. NEUMAN‡

†*Instituto de Desarrollo Tecnológico para la Industria Química (INTEC), UNL-CONICET, Güemes 3450, 3000 Santa Fe, Argentina. E-mail: tsinoli@ceride.gov.ar*
‡*Mathematics Department, Fac. Ing. Química, Universidad Nacional del Litoral, Santiago del Estero 2829, 3000 Santa Fe, Argentina. E-mail: cneuman@fiqus.unl.edu.ar*

***Abstract*— The optimal feedback control of nonlinear chemical processes, specially for regulation and set-point changing, is attacked in this paper. A novel procedure based on the Hamiltonian equations associated to a bilinear approximation of the dynamics and a quadratic cost is presented. The usual boundary-value situation for the coupled state-costate system is transformed into an initial-value problem through the solution of a generalized algebraic Riccati equation. This allows to integrate the Hamiltonian equations on-line, and to construct the feedback law by using the costate solution trajectory. Results are shown applied to a classical nonlinear chemical reactor model, and compared against standard MPC and previous versions of bilinear-quadratic strategies based on power series expansions.**

***Keywords*— process control, nonlinear dynamics, optimization, Hamiltonian systems.**

I. INTRODUCTION

A diversity of control techniques still compete on applicability and efficiency for general nonlinear processes. Since nonlinearities and main qualitative features of industrial processes are often detected without having a complete mathematical description of their dynamics, some techniques are being developed to overcome the use of models in designing control laws. Partial Control is one of these novel decentralized strategies conceived for meeting multiple economic objectives by feedback control of a few 'dominant variables' (see Tyréus, 1999). The concept is appealing because, if successful, a few SISO control loops replace the conventional process model for control purposes. Individual loops are in principle simpler to treat, instrument, and tune than multidimensional interconnected situations.

The steady-states' phase-plots for nonlinear dynamics adopt different patterns, sometimes leading to bifurcations, limit cycles, or strange attractors in high dimensions (Strogatz, 1994; Costanza, 2005a). These patterns may change, even structurally, when parameters of the dynamics vary (equilibrium control values may be regarded as parameters, specially when each manipulated variable is proportional to some physical variable like temperature or flow rate (Aris, 1999). Consequently, changing operation from one steady-state to another may imply working near bifurcation points, where model information is essential.

Disjoint from heuristic methods there exist a range of model-based approaches, Model Predictive Control (MPC) becoming the most notorious. Still, for nonlinear systems MPC is only recommended in very special situations (Figueroa, 2001; Norquay *et al.,* 1998) given the computational complexity of the calculations involved. Most successful industrial applications of MPC reported so far are in refining and petrochemical plants, where (continuous) processes are run near optimal steady-states and model linearizations are reliable approximations. Only one of the available commercial software packages was cautiously suggested for truly nonlinear or batch processes in a recent survey (Qin and Badgwell, 1997).

Some numerical implementations of MPC discretize the whole event space $\mathcal{X} \times \mathcal{T}$ from the beginning, which for nonlinear systems have predictable shortcomings (an event is a pair (x, t) of a state $x \in \mathcal{X}$ and a time instant $t \in \mathcal{T}$, $\mathcal{X}$ denoting the state space and $\mathcal{T}$ the time span under consideration. Trajectory perturbations are increasingly important in the nonlinear case, specially near unstable steady-states. Since states are allowed to take only discrete positions in the calculations, being near unstable equilibria may not be noticed by the algorithms. Contrarily, feedback laws are determined from ODE's parameters that contain all stability information. Also, control values calculated from these laws depend on the exact (actual) values of state variables. To attain the same degree of accuracy with the MPC approach involves refining the discretization (so increasing the computing time, which makes troublesome to keep on-line work), and guaranteeing convergence of this refining (rarely taken into account). Minimizing computing time in nonlinear MPC is not a trivial problem, as reflected by the va-