ADAPTIVE FILTERING USING PROJECTION ONTO CONVEX SETS

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***Abstract*— In this paper we propose a novel adaptive filtering algorithm. The algorithm exploits the information given by the power spectral density of the noise extracted from the periodogram of filtering error. The goal is try to match the spectral properties of the error filtering with the spectral properties of the measurement noise. With this in mind appropriate convex and closed sets are built and projections onto them are computed. The simulation results show that the algorithm has excellent convergence properties with a reduced number of updates. This could be exploited to obtain a lower computational load.**

***Keywords*— Adaptive Filtering, Projections, Convex Sets, Periodogram, Power Spectral Density.**

I. INTRODUCTION

The problem of adaptive filtering can be interpreted as one in which an unknown system has to be estimated. Adaptive filtering has a great number of applications such as channel equalization, noise cancellation, echo cancellation, etc. (Haykin, 2002).

Set Theoretic Estimation has received considerable attention for the last 20 years (Combettes, 1993). It has been applied to a considerable number of problems like image processing (Combettes, 1997), signal restoration (Trussell and Civanlar, 1984), etc. The idea behind this approach is to use certain *a priori* information about the object to be estimated. The solution is required to be consistent with this information. This is the only requirement to be fulfilled.

The *a priori* information is used to build sets (*property sets*), in such a way that they contain the true object with a high degree of confidence. A solution to the problem can be stated in the following manner: find one element in the intersection of the sets. This task could be very difficult to implement in practice (Combettes, 1993).

The application of this framework to adaptive filtering has been reported too. Dasgupta and Huang (1987), Gollamudi *et al.* (1998), Huang (1986) and Nagaraj *et al.* (1999) proposed to bound the *feasibility set* (the intersection set built with the sets representing the pieces of *a priori* information) with hyperellipsoids at each time instant. Yamada *et al.* (2002) utilized a method based on parallel subgradient projection (PSP) techniques onto convex sets for recursive estimation of the true system. Yukawa and Yamada (2004) proposed an interesting modification to the PSP algorithm, which improves its performance. In those previous works, information about additive noise is used for the construction of the *property sets.* The algorithms derived in those works show excellent convergence properties for highly-colored inputs and reduced number of updates.

This paper proposes a novel adaptive algorithm following the ideas given by Yamada *et al.* (2002). It uses information about the power spectral density of the noise. The periodogram of the filtering error plays a fundamental role in the algorithm for testing the consistency of the successive estimations with the information about the power spectral density of the noise.

Throughout the paper, the following notations are used: $\mathbb{R}^N$ and $\mathbb{C}^N$ are real and complex Hilbert spaces with inner products $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T\mathbf{y}$ and $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^H\mathbf{y}$ respectively, where the superscripts T and H denote transposition and complex conjugate transposition. For any nonempty closed convex set $\mathcal{C}$ in a Hilbert space $\mathcal{H}$, the *projection operator* $P_{\mathcal{C}} : \mathcal{H} \to \mathcal{C}$ is defined by $\|\mathbf{x} - P_{\mathcal{C}}(\mathbf{x})\| = \min_{\mathbf{y}\in\mathcal{C}} \|\mathbf{x} - \mathbf{y}\| \ \forall \mathbf{x} \in \mathcal{H}$.

II. PRELIMINARIES

Let $\mathbf{w}_0 = \left[w_0^0\ w_0^1 \ldots\ w_0^{N-1}\right]^T \in \mathbb{R}^N$ be an unknown linear FIR system. This is a common assumption in system identification because FIR systems constitute a simple and effective approximation in many practical problems. The associated adaptive filtering problem is shown in Fig. 1. The input signal at time n, $\mathbf{x}(n) = [x(n)\ x(n-1) \ldots\ x(n-N+1)]^T \in \mathbb{R}^N$ pass through the system giving an output $\mathbf{w}_0^T\mathbf{x}(n) \in \mathbb{R}$. This output is observed but in this process usually appears a noise $v(n) \in \mathbb{R}$ which will be considered additive. Thus, each successive input $\mathbf{x}(n)$ gives an output $y(n) = \mathbf{w}_0^T\mathbf{x}(n) + v(n)$. The idea is to find $\hat{\mathbf{w}}_{n+1}$ to estimate $\mathbf{w}_0$. This filter receives the same input $\mathbf{x}(n)$, leading to an output estimation error