A THIRD ORDER DISCRETE EVENT METHOD FOR CONTINUOUS SYSTEM SIMULATION

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***Abstract*— This paper introduces a new numerical method for integration of ordinary differential equations. Following the idea of quantization based integration, i.e., replacing the time discretization by state quantization, this new method performs a third order approximation allowing to achieve better accuracy than their first and second order predecessors. It is shown that the new algorithm satisfies the same theoretical properties of the previous methods and also shares their main advantages in the integration of discontinuous systems.**

***Keywords*— Hybrid systems, ODE integration, Discrete Event Systems.**

I. INTRODUCTION

Numerical integration of ordinary differential equations (ODEs) is a topic which has advanced significantly with the appearance of modern computers. Based on classic methods like Euler, Runge–Kutta, Adams, etc., several variable–step and implicit ODE solver methods were developed (Hairer *et al.*, 1993; Hairer and Wanner, 1991). Simultaneously, several software simulation packages have been developed implementing these algorithms. Among them, Matlab/Simulink (Shampine and Reichelt, 1997) is probably the most popular and one of the most efficient.

In spite of the several differences between the mentioned ODE solver algorithms, all of them share a property: they are based on time discretization, giving a solution obtained from a difference equation system (i.e., a discrete–time model).

A completely different approach for ODE numerical integration started to develop since the end of the 90's, in which time discretization is replaced by state variables quantization. As a result, the simulation models are not discrete time but discrete event.

The origin of this idea can be found in the definition of Quantized Systems (Zeigler and Lee, 1998). Quantized Systems were reformulated with the addition of hysteresis –to avoid the appearance of infinitely fast oscillations– and formalized as a numerical algorithm for ODE's by Kofman and Junco (2001), where the Quantized State Systems (QSS) and the QSS method were defined.

The following step was the definition of the method of second order quantized state systems (QSS2) (Kofman, 2002), and then both methods were extended to the simulation of differential algebraic equations (DAEs) (Kofman, 2003) and discontinuous systems (Kofman, 2004).

The discrete event nature of these methods make them particularly efficient in the last case, and a considerable reduction of computational costs with respect to the most sophisticated discrete time methods can be observed.

Despite their simplicity, the QSS and QSS2 methods satisfy some strong stability, convergence and error bound properties, and they intrinsically exploit sparsity in a very efficient fashion.

This paper continues the previous works by formulating the method of third order quantized state systems (QSS3) which permits improving the accuracy of QSS and QSS2 conserving their main theoretical and practical advantages. An additional advantage of QSS3 is that the choice of the quantum becomes less critical than in the lower order methods since it can be adopted in a conservative fashion without affecting considerably the number of calculations.

After a brief introduction recalling the principles of quantization based integration, the definition of the QSS3 method will be introduced. Then, we shall prove that it is *legitimate*, i.e., that it cannot produce a Zeno–like behavior, and we shall deduce the input–output relationships of the basic components of QSS3 (quantized integrators and static functions). Then, after a brief discussion about the theoretical properties of QSS3, two relatively complex simulation examples will be introduced.

II. QUANTIZATION BASED INTEGRATION

A. QSS–Method

Consider a time invariant ODE in its State Equation System (SES) representation:

$$\dot{x}(t) = f(x(t), u(t)) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector and $u(t) \in \mathbb{R}^m$ is an input vector, which is a known piecewise constant function.