

STABILITY OF SYSTEMS WITH TIME-VARYING DELAY

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Abstract— This paper presents a useful theoretical extension of the Lyapunov-Krasovskii's theory to analyse the exponential stability of differential equations with time-varying delay. The presented theoretical analysis allows establishing the coefficients of an upper exponential bound of the real response of a delayed system. In addition, we propose stability conditions –delay amplitude independent- applied to linear and non-linear systems with time-varying delay. Based on the Krasovskii-type functional, the proposed functionals incorporate information of the delayed system. The main motivation of this paper is to arrive at conditions of exponential stability that show directly the influence of the time-varying delay and the non-delayed dynamics on the real response of a delayed system. Theoretical results are tested through a numerical example.

Keywords— delayed systems, exponential stability, linear systems, non-linear systems, time-varying delay.

I. INTRODUCTION

Many real systems are represented by delayed differential equations, with models applied to biology, chemistry, economics, mechanics, medicine, physics, etc. (Hale and Lunel, 1993; Kolmanovskii and Myshkis, 1999). In general, the introduction of a time delay into the differential equations may lead to instability or to a bad performance of the system (Kolmanovskii and Myshkis, 1992; Niculescu, 2001; Richard, 2003). On the other hand, the literature presents various criteria to analyse the stability of non-linear and linear systems with time delay (Kharitonov, 1999; Kolmanovskii *et al.*, 1998 and 1999b). Main stability conditions are based on the Lyapunov-Krasovskii functional (Mao, 1996; Verriest and Niculescu 1997; Verriest and Aggoune, 1998; Tchanganani *et al.*, 1998; Kolmanovskii *et al.*, 1999a; Niculescu and Mazenc, 2001; Jafarov, 2001; Nian, 2003; Han, 2004; Pepe, 2005); matrix measures norm (Mori, 1985, Niculescu *et al.*, 1995); and Razumikhin functions (Jankovic, 1999; Teel, 1998), but these papers don't analyse how to modify the transitory of the system, which is useful for the design and control of real systems. The general purpose of this paper is to analyse the exponential stability of systems with time-varying delay studying how some factors (for example, time delay) establish the stability regions and modify

the response of such systems.

Based on the Lyapunov-Krasovskii's theory applied to delayed differential equations and the Lyapunov's theory applied to differential equations, we propose a useful theoretical extension to analyse the exponential stability of delayed differential equations establishing an upper bound of the real response of a delayed system. In addition, we propose sufficient conditions –independent of the delay amplitude- to analyse the exponential stability of linear and non-linear systems with time-varying delay. In general, the used functionals are based on the Krasovskii-type functional and they depend on setting positive definite symmetric matrices in the form of Riccati equations, which are generally set by trial and error. In this work, the proposed functionals incorporate information of the delayed system arriving at useful stability conditions for control of systems. Such conditions show the influence of the time-varying delay and the non-delayed dynamics (which can be generally handled) on the stability and response of a delayed system. The paper also includes a numerical example that helps the reader to test the reached theoretical results.

The paper is organized as follows. Section II gives the notation used in this paper. In section III some background materials on delayed differential equations and the Lyapunov-Krasovskii's theory are introduced. Section IV presents the proof of the proposed main theorem for exponential stability of systems with time-varying delay. The proposed conditions of exponential stability applied to linear and non-linear systems with time varying delay are proven in section V. In addition, the stability region is analysed. Section VI presents a numerical example to verify the reached theoretical results. Finally, general conclusions are given in section VII.

II. NOTATION

In this paper, the following notation is used: $h(t) \in \mathfrak{R}^+$ denotes the time delay. We assume that the delay is finite and $\dot{h}(t) < 1$. Here, $x(t) \in \mathfrak{R}^n$ and $\|x\|$ is the Euclidean norm of x . If B is a matrix or vector then B^T is the transpose of B , $|A|$ denotes the Euclidean norm of $A \in \mathfrak{R}^{m \times n}$ defined by $|A| = \sup_{\|x\|=1} \|Ax\|$, $\mu(A)$ denotes the matrix measure (derived from the Euclidean norm) of