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UNIVERSAL CONSTRUCTION OF CONTROL LYAPUNOV FUNCTIONS FOR LINEAR SYSTEMS *

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Abstract– This paper develops a method by which control Lyapunov functions of linear systems can be constructed systematically. It proves that the method can provide all quadratic control Lyapunov functions for a given linear system. By using the control Lyapunov function a linear feedback is established to stabilize the linear system. Moreover, it can also assign poles of the closed-loop system in the position designed in advance.

Keywords– linear systems, stabilization, control Lyapunov functions

I. INTRODUCTION

In the early days of control theory investigation, most of concepts such as stability, optimality and uncertainty were descriptive rather than constructive. The situation has been gradually changed in the last two decades. Kokotovic and Arcak (2001) made a survey for the alteration and call it 'activation'. A prominent example of the activation is the concept of control Lyapunov function (henceforth CLF for short). Traditionally, Lyapunov function is a powerful tool to the analysis of stability of dynamic systems. Artstein (1983) and Sontag (1983) considered respectively the stabilization of control systems and extended the notion of Lyapunov function to that of control Lyapunov function. It has been verified that a nonlinear system can be stabilized by a relaxed state feedback if and only if it holds a CLF (Artstein 1983). Moreover, Sontag (1989) dealt with the stabilization of affine systems and presented a universal feedback scheme by using CLF. These achievements greatly motivated the investigation of CLF, and CLF were widely adopted in various design problems. For instance, from Freeman and Primbs (1996), Freeman and Kokotovic (1996) Liberzon *et al.* (2002), Cai and Han (2005), Sepulchre *et al.*, (1997), the readers can find many meaningful results.

However, being similar to the situation of Lyapunov function, a CLF is not always available for a given system, even for a linear system. We have no a general method to construct a CLF. Hence, the construction of CLF becomes the bottleneck of the design technique developed by using CLF.

This paper presents a systematic study for CLF of linear systems. We give the necessary and sufficient conditions for CLF of linear systems. We establish a method to construct a quadratic CLF for a linear system by solving a Lyapunov equation. Freeman and Primbs (1996) also gave an approach to obtain a CLF for a linear system by solving a Riccati equation. It is clear that Lyapunov equation is much simpler than Riccati equation since the former is linear and the later is quadratic. We then prove that for a linear system there exists a quadratic CLF if it has a CLF. A linear feedback by CLF is designed to stabilize the given system.

The significance of the CLF comes from the universal formulas. After the works of Sontag (1989), there are a number of universal feedback schemes presented for the stabilization, tracing, regulation, robust control, optimization and so on. A toolbox for the design using CLF is then easily developed. It means that compensators for the design problems mentioned above can be achieved if a CLF is available. These design techniques can be applied to a linear system if a CLF is obtained although it will lead to a nonlinear system. Another significance of the investigation of CLF of linear systems is that it may open a way to the construction of CLF for affine systems by the zero dynamic method (Isidori 1989) and for the general nonlinear systems by the central

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