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ANALYTIC SOLUTION FOR THE INDENTATION OF A TRANSVERSELY ISOTROPIC ELASTIC LAYER BONDED TO A RIGID FOUNDATION

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Abstract— **The analytical solution for a transversely isotropic linear elastic layer bonded to a rigid foundation and indented by a rigid cylinder or sphere was developed. This solution follows procedures used by others to solve contact problems of linear anisotropic materials. The solution can be used to find the stress distribution and the displacement field in anisotropic layers like articular cartilage. The solution was used to compare stresses and displacements in articular cartilage assuming two sets of engineering properties with different degrees of anisotropy. The results may support current research about the relation between impact loading on the articular cartilage and the development of osteoarthrosis.**

Keywords— **Indentation, Transversely isotropic, articular cartilage, osteoarthrosis.**

I. INTRODUCTION

Articular cartilage is the material covering the end of the bones inside the synovial joints. It has excelent mechanical properties to transmit loads and to allow relative movements without significant wear (Mow *et al*., 1980). In some cases, articular cartilage begins to deteriorate and the underlying bone grows until direct contact bone to bone is produced inside the joint causing inmobilization and severe pain. This desease, known as osteoathrosis, can lead to considerable hospital stays and extended periods of lost work days (Mackenzie *et al.,* 1988).

Significant research has been undertaken to understand the ethiology of osteoartrosis (Ewers *et al.* 1998). Some authors have suggested that osteoarthrosis is due to impact loading to the joint and in vivo experiments in animals have been used to study this correlation (Newberry *et al.,* 1998). The most widely used mechanical model for the articular cartilage is the biphasic (Mow *et al*., 1980), which considers the tissue composed of a solid phase and a fluid. Under impact loading and equilibrium, biphasic cartilage can be analyzed as an equivalent elastic layer (García *et al.,* 1998). It has been shown that an isotropic model for articular cartilage is unable to simulate the response in indentation experiments (Mow *et al.,* 1989). On the other hand, a transversely isotropic model in which the Young´s modulus in the plane of the cartilage (plane of isotropy) is higher than that in the direction of the loading, provides a good fit to the experimental curves (Cohen *et al.*, 1993; García *et al.*, 2000).

In situ indentation tests with spherical and cylindrical indenters have been widely used to determine the elastic properties of articular cartilage (Töyräs *et al.*, 2001), which are necessary to asses the condition of the tissue in animal experiments and to undertake finite element analysis of the joints. If a transversely isotropic model is adopted for the articular cartilage, there is no analytical elastic solution for this layer firmly bonded to the rigid foundation. In this study, the analytical solution for the indentation of a transversely isotropic elastic layer bonded to a rigid foundation was developed based on the general equations of the anisotropic elasticity presented by Lekhnitskii (1981) and procedures followed by others to solve linear elastic contact problems (Sakamoto *et al.*, 1991). This solution may help to analyze articular cartilage under impact loading and to develop procedures to determine their elastic constants from in situ tests.

II. METHODS

A. Problem formulation

The model to be solved consists of a transversely isotropic linear elastic layer bonded to a rigid foundation and indented by a cylindrical or spherical punch. Isotropic planes are perpendicular to z-axis (Fig. 1), which is the axial symmetry axis. A cylindrical coordinate system was used with the origin located at the intersection between the rigid foundation and the symmetry axis. The material is characterized by five elastic constants, which are the Young's modulus and Poisson's ratio (E, ν) in the isotropy plane and the Young's modulus, shear's modulus and Poisson's ratio out of the isotropy plane (E', ν' and G'). These engineering constants can also be related to the elasticity coefficients C_{11}, C_{33}, C_{44}, C_{13} and C_{12} used by Lekhnitskii (1981) as shown in Appendix A.

This model can be used to represent the mechanical behavior of articular cartilage, firmly attached to the