FREQUENCY DOMAIN APPROACH TO HOPF BIFURCATION FOR VAN DER POL EQUATION WITH DISTRIBUTED DELAY

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Abstract— **The van der Pol equation with a distributed time delay and a strong kernel is analyzed. Its linear stability is investigated by employing the generalized Nyquist stability and Routh–Hurwitz criteria. Moreover, local asymptotic stability conditions are also derived in the case of the strong kernel. By using the mean time delay as a bifurcation parameter, the model is found to undergo a sequence of Hopf bifurcations. The direction and the stability criteria of the bifurcating periodic solutions are obtained by the graphical Hopf bifurcation theory. Some numerical simulation examples for justifying the theoretical analysis are also given.**

Keywords— **Van der Pol equation, distributed delay, Hopf bifurcation, periodic solutions, a polycyclic configuration.**

I. Introduction

The classical van der Pol equation, which describes the oscillations in a vacuum tube circuit, is the second-order nonlinear damped system governed by

$$\begin{cases} \dot{x}_1^*(t) = x_2^*(t) - f[x_1^*(t)] \\ \dot{x}_2^* = -x_1^*(t) \end{cases}, \tag{1}$$

where $f(x) = ax + bx^3$.

System (1) is considered as one of the most intensely studied systems in nonlinear dynamics (Guckenheimer and Holmes, 1983) and has served as a basic model of self-excited oscillations in physics, electronics, biology, neurology and other disciplines. Many efforts have been made to find its approximate solutions (Buonomo, 1998; Frey and Douglas, 1998; Guckenheimer and Holmes, 1983; Venkatasubramanian and Vaithianathan, 1994) or to construct simple maps that qualitatively describe the important features of its dynamics. Hence, if $a > 0$ and $b > 0$ in $f(x)$, the origin is globally asymptotically stable and so there is no periodic solution for system (1). However, if $a < 0$ and $b > 0$, a unique stable periodic solution does exist.

It would be very useful if we have some knowledge about the existence of periodic solutions for delay nonlinear differential equations. As we know, in ordinary differential equations, one of the simple ways in which a non-constant periodic solution can arise is through Hopf bifurcation. This occurs when two eigenvalues cross the imaginary axis from left to right as a real parameter in the equation passes through a critical value (Gopalsamy, 1992; Hale and Verduyn, 1993; Hassard *et al*., 1981; Iooss and Joseph, 1989; Kung, 1992; MacDonald, 1989). In a study of classical van der Pol oscillators, it has been shown that oscillations occur when a stable equilibrium undergoes the singularity induced bifurcation in the slow differential-algebraic model, which, in turn, corresponds to the occurrence of supercritical Hopf bifurcations in the singularly perturbed models (Venkatasubramanian and Vaithianathan, 1994). Frey and Douglas (1998) proposed a class of relaxation algorithms for finding the periodic steady-state solution of a van der Pol oscillation. Buonomo (1998) gave the periodic solution of the van der Pol equation in the form of a series converging for all values of the damping parameter. Recently, discrete time delay was introduced into system (1) and the following pair of delay differential equations obtained (Murakaimi, 1999).

$$\begin{cases} \dot{x}_1^*(t) = x_2^*(t-\tau) - f(x_1^*(t-\tau)) \\ \dot{x}_2^*(t) = -x_1^*(t-\tau) \end{cases}. \tag{2}$$