## CONTROL OF THE REACHING MODE IN VARIABLE STRUCTURE SYSTEMS

R. J. MANTZ<sup>†</sup>, H. DE BATTISTA<sup>‡</sup>, F. D. BIANCHI<sup>‡</sup> and R. D. FERNÁNDEZ\*

LEICI, Dto. de Electrotecnia, Facultad de Ingeniería, UNLP, C.C.91 (1900) La Plata, Argentina † UNLP-CICpBA, mantz@fisica.unlp.edu.ar † UNLP-CONICET, deba@fisica.unlp.edu.ar, fbianchi@volta.ing.edu.ar \* UNLP-UNPSJB, rfcrnand@fisica.unlp.edu.ar

Abstract— This paper focuses on the behaviour of variable structure systems with dynamic control, particularly during the reaching mode of operation. It is shown that stability problems may arise during this reaching phase. The causes of these problems are closely related with the problems of windup commonly found in conventional control systems with actuator constraints. Methods for stabilization of the reaching mode are proposed which are based on the concepts of 'realizable reference' and observers. Well-known algorithms that have been previously proposed from empiric ideas, can now be rigorously derived using these concepts. The theoretical framework developed by Kothare and co-workers in the context of windup is generalized to study and design control algorithms for the reaching mode.

*Keywords*— reaching mode, sliding mode, variable structure systems, windup.

## I. INTRODUCTION

It is well-known that variable structure systems (VSS) undergoing sliding motions are robust to parameter uncertainties and external disturbances. Moreover, the order of the dynamic system is reduced during the sliding mode, and the sliding dynamics becomes dependent on the designer-chosen sliding surface (Utkin, 1978; Sira-Ramírez, 1988, 1996; Hung et al., 1993).

Actually, the complete response of a VSS comprises two phases or operating modes: the reaching mode (RM) and the sliding mode (SM). Even though the latter has been more discussed in literature, the former is not less important when the global performance is considered. Different approaches to the RM problem can be found in Hung et al. (1993). Despite their outstanding contributions, these approaches do not focus on the state dynamics (Hung et al., 1993; Mantz et al., 2001) and, in general, are particular or intuition-based solutions.

This paper studies a particular behaviour that may lead to a serious degradation and, moreover, instability of the RM, deteriorating the global performance of the VSS. The work puts special emphasis on VSS with dynamic controllers where this undesirable behaviour is more evident. This degradation of the RM is linked in the paper to another problem extensively studied in the last years: windup (Fertik and Ross, 1967; Doyle et al., 1987; Astrom and Rundqwist, 1989; Peng et al., 1996; Romanchuk, 1999; Wu and Grigoriadis, 1999). Based on this connection between both problems, different methods of RM compensation are proposed. They make use of the concepts of realizable references and observers. Moreover, the unified theoretical framework proposed by Khotare et al. (1994) to address the problem of windup is generalized to solve the RM problem in VSS.

In the following section, the problem is posed and illustrated through an example. Then, the similarities among windup and RM problems are stressed. In subsection II.B., a pair of RM compensation algorithms based on the concepts of realizable references and observers are derived. At the end of the section, the framework developed by Khotare and co-workers is generalized to address the RM problem. Finally, the conclusions of the paper are summarized.

## II. PROBLEM FORMULATION AND MAIN RESULTS

## A. Problem formulation

Figure 1 shows a schematic diagram of a variable structure controlled system. P is the process to control. The block  $\Delta$  takes into consideration the parametric uncertainties. The switch L is driven by the output of the controller K, namely s(x). It is assumed that K may include a dynamic expansion to reject steady state disturbances (Utkin, 1999) (obviously, it is not possible to include in K a dynamic expansion to reduce chattering problems (Sira-Ramírez, 1993), which must be inserted at the input of P). Then, s(x) depends on the state variables of the process  $(x_s)$  and of  $K(x_k)$ . It is also assumed that K is an LTI system, which is an usual election in most applications (some non-linear process functions  $y = f(x_s)$  can be explicitly chosen as inputs of K to consider the case of non-linear processes). Then,  $K(s) = C(sI - A)^{-1}B + D$ . Hereinafter,