STABILITY ANALYSIS OF A CERTAIN CLASS OF TIME-VARYING HYBRID DYNAMICAL SYSTEMS

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Abstract— **In this work we study the exponential stability of a class of hybrid dynamical systems that comprises the sampled-data systems consisting of the interconnection of a time-varying nonlinear continuous-time plant and a time-varying nonlinear discrete-time controller, assuming that the sampling periods are not necessarily constant. For this purpose we develop an Indirect Lyapunov Method of analysis, and show that under adequate hypotheses the exponential stability of the hybrid dynamical system is equivalent to the exponential stability of its linearization.**

Keywords— **Sampled-data systems; Lyapunov stability; Hybrid systems; Discrete-time systems.**

I. INTRODUCTION

Sampled-data control systems consisting of the interconnection of a continuous-time nonlinear plant (described by a system of non autonomous first order ordinary differential equations) and a nonlinear digital controller (described by a time-varying system of first order difference equations) is a hybrid system, in the sense that some of its variables evolve smoothly in continuous time while the others change only in a discrete set of time instants. The coexistence of these two different time scales makes it hard to analyze the stability properties of this kind of systems.

Stability analysis of sampled-data systems in the whole time scale was primarily studied for linear systems (Francis and Georgiou, 1988; Iglesias, 1994). The nonlinear case was addressed in the recent papers (Hou *et al.*, 1997; Mancilla-Aguilar *et al.*, 2000; Hu and Michel, 2000a, 2000b), in which qualitative properties of sampled-data control systems, where the plant and the controller are time-invariant, were obtained.

In (Mancilla Aguilar *et al.*, 2000), in particular, we analyzed the stability of a class of hybrid dynamical systems, those described by time-invariant *hybrid equations*, that contains as particular cases the interconnected system consisting of a continuous time plant and a digital controller and that of a continuous time plant and a certain class of hybrid controllers presented in (Rui *et al.*, 1997). In this work we consider the class of hybrid systems described by time-varying hybrid equations with non regular sampling times, and extend some results of (Mancilla Aguilar *et al.*, 2000) to the class of hybrid systems described by this type of equations. To be more precise, we obtain necessary and sufficient conditions for the exponential stability of the system in terms of its linearization, developing in this way an *Indirect Lyapunov Method* for this kind of dynamical systems. Our results can be straightforwardly applied to the study of the exponential stability of sampled-data systems whose plant and controller are nonlinear and time-varying, as these systems are a particular class of those under study. They also justify the method of design of nonlinear digital local exponential stabilizers based on the discretized model of the linearization of the original nonlinear control system (a more complete treatement of these topics can be found in (Mancilla Aguilar, 2001)). Two reasons motivated the treatment of non-regular sampling. The first is the uncertainty that could appear in the frequency of the sampler oscillator, that in certain applications should be of importance. The second is that our results can also be applied to the stability analysis of certain classes of switched systems whose switching times are not necessarily regularly spaced.

The paper is organized as follows. In section II we establish some notation and state the main result of the paper. In section III we present some results about the exponential stability of a perturbed discrete-time time-varying linear system. In section IV we use these results to obtain stability properties of perturbed hybrid linear systems, that enable us to develop the Indirect Lyapunov Method already mentioned, and prove the main result of the paper. Finally, in section V we present some conclusions.

II. NOTATION AND MAIN RESULT

First, we introduce some notation that will be used in this work.

Let $\mathbb{N}_0$, $\mathbb{R}$ and $\mathbb{R}^+$ the sets of non-negative integer, real and non-negative real numbers respectively. We consider the real q-space $\mathbb{R}^q$ as a normed space, with