ON THE EXISTENCE OF NORM-ESTIMATORS FOR SWITCHED SYSTEMS

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***Abstract*— In this paper, we prove the existence of norm-estimators for switched nonlinear systems. The proof is based on an existing converse Lyapunov theorem for IOSS nonlinear systems, and on the association of the switched system with a nonlinear system with inputs and disturbances that take values in a compact set.**

***Keywords*— Norm estimators, Converse Lyapunov theorems, IOSS, Nonlinear switched systems.**

I. INTRODUCTION

Recently, the study of switched systems has received a great deal of attention, being the rapidly developing area of intelligent control an important source of motivation for this study. Informally, a switched system is a family of continuous-time dynamical subsystems and a rule that determines the switching between them. The recent paper (Liberzon and Morse, 1999) is a very interesting survey on the subject, where an updated account of results and open problems may be found.

This paper concerns itself with the following question, for a switched system: *is it possible to estimate, based on external information provided by past input and output signals,* the magnitude *of the internal state* $x(t)$ *at time* t?

State estimation is central to control theory as it arises in signal processing applications (Kalman filters), in stabilization based on partial information (observers), etc. An open question is the derivation of useful necessary and sufficient conditions for the existence of observers, *i. e.*, dynamical systems which provide an estimate $\hat{x}(t)$ which converges to the state $x(t)$ of the system of interest, using the information provided by the sets of past inputs and outputs, $\{u(\tau)\,,\,\tau \le t\}$ and $\{y(\tau)\,,\,\tau \le t\}$ respectively. In order to stabilize a system to an equilibrium (that we will assume with no loss of generality to be the origin) of an Euclidean space, it may suffice to have a norm-estimate, that is to say, an upper bound $\hat{x}(t)$ on the *magnitude* (norm) $|x(t)|$ of the state $x(t)$. Indeed, it is often the case (Jiang and Praly, 1992; Praly and Wang, 1996) that norm-estimates suffice for control applications. To be more precise, in the context of switched systems, one wishes that $\hat{x}(t)$ becomes an upper bound of $|x(t)|$ as $t \to \infty$ uniformly with respect to the switching signal (see next section for precise definitions). We are thus interested in *uniform norm-estimators* which, when driven by the input-output data generated by a switched system, produce such an upper bound $\hat{x}(t)$ irrespectively of the switching signal.

One obvious necessary property for the possibility of norm-estimation is that the origin must be uniformly (with respect to the switching signal) globally asymptotically stable with respect to the "subsystem" consisting of those states for which the input $u \equiv 0$ produces the output $y \equiv 0$. In this case the switched system is *uniformly zero-detectable.* However this property is not sufficient, since one should ask that, irrespectively of the switching signal, when inputs and outputs are small, states should also be small, and if inputs and outputs converge to zero as $t \to \infty$, states do too.

On the other hand, the notion of input-output-to-state stability (IOSS), introduced by Sontag and Wang (1997) for a system

$$\left\{ \begin{array}{rcl} \dot{x} & = & f(x,u) \\ y & = & h(x), \end{array} \right. \tag{1}$$

resulted in an useful paradigm in the study of nonlinear detectability. In that paper the authors describe relationships between the existence of full state observers and the IOSS property.

In a recent paper Krichman *et al.* (2001) proved that system (1) is IOSS if and only if it admits a norm estimator (in a sense that will be made precise in Section 4). This result suggests that, for our purposes, the right notion of detectability is the generalization of the IOSS property to switched systems.

Since in (Krichman *et al.*, 2001) it was also shown that this result is, in turn, a consequence of a necessary and sufficient characterization of the IOSS property in terms of a smooth dissipation function (an uniform IOSS Lyapunov function), the problem that naturally appears in our context may be stated as follows: given a switched system with outputs whose component subsystems are each IOSS, find necessary and sufficient