

## GROEBNER BASES FOR DESIGNING DYNAMICAL SYSTEMS

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**Abstract**— The design or synthesis of systems exhibiting a prescribed trajectory is presented in this paper. The design process is based on algebraic concepts, and it relies heavily on the use of Groebner bases. It is assumed that both the trajectory and its dynamics can be represented as algebraic relationships between the variables of the system and their first derivatives. The method yields a dynamical systems with the desired behavior as one of its many solutions.

**Keywords**— Groebner bases - control theory - dynamical systems - differential equations.

## I. INTRODUCTION

Over the last years, a great progress has been performed in the field of differential equations through the study of the different dynamical configurations obtained when varying some parameters of the system. In this field, one of the most promising results is the representation of certain dynamic phenomena using an elementary form, with polynomial-type relationships among their variables, the normal form. This representation describes the typical effects in the simplest way not only among the main variables (normal form) but also its parameters (unfoldings), after making a series of transformations on the original system.

The normal form and its unfoldings in the parameter space explains the elementary dynamics exhibited by the system, and this model is still valid for certain perturbations in higher order terms (*i.e.*, relationships between polynomials of higher order). Synthesizing dynamical systems by means of normal forms became feasible with the appearance of algebraic symbolic algorithms, contributing to study the system dynamics in an analytic way. Although in the majority of the nonlinear systems it is not possible to find explicit solutions, except through the use of numerical methods, these techniques are still very powerful for validating the results.

Advances in computer technology and the availability of software packages to perform symbolic mathe-

matics, like Mathematica, Maple, Reduce, etc. stimulated the study of some theories and techniques developed previously, particularly the concept of Groebner bases, formulated by Buchberger in 1965. (A brief review of the algebraic concepts used through the paper is contained in the Appendix.) Buchberger proved the fundamental theorems on which the theory is based and proposed an algorithm to compute such bases (Buchberger, 1965, 1970, 1995). The advent of specialized software for computational algebra (CoCoA, Macaulay and Singular, etc.) expanded the studies of computational algebra to other fields, for example dynamical systems and control theory (Forsman, 1995; Fortell, 1995; Jirstrand, 1996; Alwash, 1996, among others).

The design of dynamical systems exhibiting a prescribed orbit as one of its solutions is explored in this paper. The design process consists in finding a differential system from the specification of the desired orbit and its dynamical behavior by means of Groebner bases. It seems that normal forms—a polynomial characterization of the system dynamics—and Groebner bases will develop profound connections in a near future. In this vein, this paper constitutes a first step in enlightening some preliminary applications of synthesis of nonlinear systems using Groebner bases.

## II. OBJECTIVE

The synthesis process developed in this paper aims at designing a system  $\dot{x} = f(x)$  with an *a priori* chosen solution  $x = x_d(t)$ , and also endowing this solution with asymptotically orbital stability. The proposed solution defines a trajectory or orbit, *i.e.* the image  $\gamma$  of  $x_d(t)$  in the state-space,

$$\gamma = \{x \in \mathbb{R}^n \mid x = x_d(t), t \geq 0\}.$$

This set is positively invariant, and the dynamical behavior of the solution is determined by the restriction of  $f(\cdot)$  onto the set  $\gamma$ , given by the time derivative of the solution  $\dot{x}_d(t) = f(x_d(t))$ . In other words, we will impose that every solution with an initial condition in  $\gamma$ , remains in it for all  $t \geq 0$ . We will also require