STABILITY ANALYSIS OF DEGENERATE HOPF BIFURCATIONS FOR DISCRETE-TIME SYSTEMS

M. B. D'AMICO*, E. E. PAOLINI and J. L. MOIOLA*[1]

Departamento de Ingeniería Eléctrica y de Computadoras, Universidad Nacional del Sur
Avda. Alem 1253 - B8000CPB Bahía Blanca, Argentina
mbdamico@criba.edu.ar
* *CONICET (Consejo Nacional de Investigaciones Científicas y Técnicas)*
[1] *Mathematical Institute, University of Cologne, 50931 Cologne, Germany*

***Abstract*— A methodology for the stability analysis of invariant cycles emerging from Hopf bifurcations in discrete-time nonlinear systems is presented. The technique is formulated in the so-called frequency-domain and it is based on the Nyquist stability criterion and a higher-order harmonic balance method. The study of a planar cubic map is included for illustration.**

***Keywords*— Hopf bifurcation, discrete-time nonlinear systems, frequency-domain, harmonic balance method, stability index.**

I. INTRODUCTION

The Hopf bifurcation theorem (HBT) for maps describes the appearance of an invariant cycle when one parameter of the system is varied appropriately. Assuming that the fixed point changes its stability, the emerging bifurcation can be supercritical or subcritical denoting the birth of stable or unstable cycles for parameter values larger or smaller than the critical one, respectively. This behavior is similar to that observed in continuous-time nonlinear systems as well as in time-delayed nonlinear systems. Consequently, a technique formulated in the frequency-domain for single-input single-output (SISO) and multiple-input multiple-output (MIMO) discrete-time systems has been introduced in D'Amico *et al.* (2002) to deal with this characteristic bifurcation. The formulas capture the dynamical behavior of the emerging invariant cycle using concepts from control theory and a second-order harmonic balance method. These results are extensions of the earlier developments obtained by Allwright (1977), and Mees and Chua (1979) for continuous-time systems.

To have a better approximation of the cycle or the possibility of studying more complex dynamical structures, it is necessary to use higher-order expansions of the classical Hopf normal form. This extension is analogous to consider the higher-order approximations obtained using the harmonic balance method in the frequency-domain. However, some care should be exercised when this result is applied to the discrete-time case (Robinson, 1999) as the nonlinear maps frequently exhibit additional dynamical phenomena, such as weak and/or strong resonances. We will not address this issue on this paper, and we will concentrate on deriving the higher-order approximation of the emerging invariant cycle, and on developing algebraic expressions of the so-called stability indices to establish the stability of the cycle even in degenerate Hopf bifurcations (Iooss, 1979; Shilnikov *et al.*, 2001). These indices allows the comparison of the results obtained via the frequency-domain approach with those given by the classical normal form method (Whitley, 1983; Glendinning, 1994; Balibrea and Valverde, 1999). The approximation of the invariant cycle is based on a higher-order harmonic balance resembling the procedures followed by Mees (1981) and Moiola and Chen (1996) for continuous-time systems.

The conditions for detecting degenerate Hopf bifurcations may be translated to the discrete-time case using the frequency-domain approach. Moreover, some of the results can be applied to a much more complex theoretical construction, such as the Poincaré map (Kuznetsov, 1995), to study the stability of quasiperiodic motion in continuous-time nonlinear circuits and systems (see, for instance, Bi and Yu, 1999).

The paper is organized as follows. In Section II, higher-order formulas to determine the stability of the invariant cycle emerging from a Hopf bifurcation are derived. The study of a planar cubic map near a degenerate condition is presented in Section III. Finally, in Section IV some conclusions are given.

II. HOPF BIFURCATION IN THE FREQUENCY-DOMAIN

Let us consider the discrete-time nonlinear system

$$x_{k+1} = Ax_k + Bg\left(Cx_k; \mu\right), \tag{1}$$