AN OUTPUT FEEDBACK ALGORITHM FOR TRAJECTORY TRACKING IN CONTROL AFFINE NONLINEAR SYSTEMS

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***Abstract*— In this work we present an output feedback algorithm that solves the trajectory tracking problem in control affine nonlinear systems. This algorithm, is an improvement, for this class of systems, of that of (Mancilla Aguilar *et al.* 2000a), since it reduces the chattering effect on the control while keeping the original performance. In addition, and via a high gain observer, it deals with discrete output measurements instead of the states, as the original algorithm does.**

***Keywords*— Sampled-data; Chattering; Observer; Output feedback; Trajectory Tracking.**

I. INTRODUCTION

Nowadays, the use of digital computers for the control of continuous-time systems is commonplace. For this reason it is important to study the digital implementation of continuous-time control laws. In this case it is assumed that the states (or outputs) of the system to be controlled are available only at certain times (sampled-data). Since the control is based on these sampled-data, during the intersample periods the control actions applied to the system will be open-loop ones, even if the original continuous-time control law is a feedback one. It is then natural to study the digital implementation of the diverse continuous-time control laws for nonlinear systems, in particular stabilizing or, more generally, trajectory tracking control laws.

Although good results are obtained via the digital implementation via Sample and Zero Order Hold (SZH) of stabilizing laws (see (Mancilla Aguilar *et al.*, 2000b) and the references therein for details), this is not the case for trajectory tracking unless strong assumptions about the tracking control law are made. An example presented in (Mancilla Aguilar *et al.*, 2000a) shows that there is no reason to expect a "nice" behavior of the implementation via SZH of a trajectory tracking control feedback law.

The algorithm proposed in (Mancilla Aguilar *et al.*, 2000a), from now on *Algorithm 0*, for a rather general class of systems, solves this problem by making the system to follow the trajectories of a model instead of discretizing the continuous-time tracking law. The control law so obtained assures semiglobal practical stability of the tracking error, with final error arbitrarily small for a small enough sampling period. The controller is robust with respect to external disturbances and actuator and data measurements errors, when all of them are small enough.

One of the main drawbacks of Algorithm 0, when applied to control affine systems, appears in the implementation. In fact, as the algorithm is based in a maximization process, when the control space is a polytope, the extremal control values appear at the vertexes. This fact usually gives origin to a *chattering effect* : the values of the control switch undesirably fast. Another drawback of the algorithm is that it makes use of the states of the system, and in general only discrete-time samples of the output are available.

In this work we develop, for control affine systems, a trajectory tracking algorithm that reduces the chattering effect appearing in Algorithm 0, while it keeps the original tracking error performance. It works with the output sampled-data due to the addition of a high gain observer developed by García *et al.*, (2000), that performs the state estimation.

The paper is organized as follows. In section II we introduce some basic definitions; in section III Algorithm 0 (adapted to control affine systems) is presented and an example where the chattering effect can be observed is shown. In section IV we present a modified algorithm that obtains a high reduction of the chattering effect, while a small tracking error is kept. In section V we review a high gain observer introduced in (García *et al.* 2000) and present the output feedback algorithm that copes with the two drawbacks mentioned above. In the same section, we apply this algorithm to the example of section III. Finally, the conclusions are presented in section VI.