DYNAMIC RIGHT COPRIME FACTORIZATION AND OBSERVER DESIGN FOR NONLINEAR SYSTEMS

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Abstract – **The output behavior of a nonlinear control system depends not only on its input but also on its initial conditions. These two factors have to be considered simultaneously in nonlinear systems design. This paper presents a new definition, called dynamic right factorization, for nonlinear dynamic control systems. This factorization takes the initial conditions as well as the system input into account. Coprimeness and fundamental properties of dynamic right coprime factorization are investigated. Its relations to the system observability and the observer design problem are discussed. An example is given to illustrate the procedure of obtaining the dynamic right factorization and designing an observer for a given nonlinear dynamic system.**

Keywords – **nonlinear system; initial condition; right coprime factorization; observer.**

I. INTRODUCTION

In the operator-theoretic approach to the study of control systems, a system Σ is considered as a mapping *P* from its input space *U* to its output space *Y*, i.e., $\Sigma\ P: U \to Y$. *P* is called the input-output mapping of Σ. Since most control systems are dynamic systems whose output behavior depends not only on its inputs but also on its states, thereby relying on the initial conditions of the systems. Precisely, the mapping of a dynamic system should be defined from a Cartesian space $X_0 \times U$ to *Y*, i.e.,

$$\Sigma \begin{cases} P: X_0 \times U \to Y \\ \quad (x_0, u) \mapsto y \end{cases} \tag{1.1}$$

where X_0 is the linear space of initial states associated with the system.

Initial states are considered, even in linear control systems, and are manipulated independently of system inputs. For linear control systems, by the linearity, we have

$$P(x_0,u) = P(x_0,0) + P(0,u) = T(x_0) + G(u)\,, \tag{1.2}$$

where the operators *T* and *G* are defined by $T: X_0 \to Y$; $T(x_0) = P(x_0,0)$ and $G: U \to Y$; $G(u) = P(0,u)$. Equation (1.2) implies that the effects of X_0 and *U* can be separately considered.

For a nonlinear control system, however, this separation does not hold in general. In the nonlinear case, more often than not, the initial state and the input have to be considered simultaneously. It is inappropriate to fix the initial state in the design and analysis for a nonlinear control system, since the dynamic behavior of a nonlinear system strongly depends on its initial conditions. For example, the systems may be stable for initial states within a set of the initial space X_0, but unstable elsewhere.

In the operator-theoretic approach to control systems, coprime factorization is one of the existing efficient methods for analysis and design (Wolovich, 1974; Kailath, 1980; Vidyasagar, 1985; Youla *et al*., 1976). By applying the operator factorization methodology, one can introduce the so-called quasi-state space, a framework similar to the state space for linear systems, for nonlinear control systems.

Nevertheless, since 1980's, the mathematical nonlinear operator theory has been introduced to design, analysis, stabilization and optimization of nonlinear control systems (see, for example, (Banos, 1994; Chen & Han, 1998; Chen & Figueiredo, 1992; Desoer & Kabuli, 1988; Hammer, 1987, 1994; Han & Rao, 1995; Paice, *et al*., 1993; Sontag, 1989; Verma & Hunt, 1993) and the references therein). However, most papers only consider the operator $\Sigma\ P: U \to Y$. The initial condition seems to be left out so that these conclusions seriously restrict the application of the operator factorization method in observability analysis and observer design for nonlinear control systems.

The present paper attempts to tackle the observer design problem for nonlinear control systems, by taking the system initial conditions into consideration, from the operator factorization approach. For this purpose, a new concept of dynamic right coprime factorization for nonlinear operators is first introduced, which is defined with respect to initial conditions. Basic properties of the dynamic right coprime factorization are then discussed. The dynamic right coprime factorization is finally applied to the problem of observer design for nonlinear control systems.

The rest of the paper is organized as follows. The

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