

NOTE

CHAOS STABILIZATION: AN INVERSE OPTIMAL CONTROL APPROACH

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Abstract— In this paper, an efficient approach is developed for global asymptotic stabilization of Chen's chaotic system, which in principle works for other complex nonlinear systems as well. Based on a recently introduced methodology of inverse optimal control for nonlinear systems, a very simple stabilization control law is derived for the desired global asymptotic stabilization. Computer simulation is given for illustration and verification.

Keywords— Chaos Control, Lyapunov Method, Inverse Optimal Control Approach

I. INTRODUCTION

Chaotic systems have been studied for quite a long time in the mathematical and physical communities, and controlling this kind of complex dynamical systems has recently attracted a great deal of attention within the engineering society. Different techniques have been proposed to achieve chaos control. For instance, linear state space feedback (Chen and Dong, 1993), Lyapunov function methods (Nijmeijer and Berghuis, 1995), adaptive control (Zeng and Singh, 1997) and bang-bang control (Vincent and Yu, 1991), among many others (Chen and Dong, 1998).

On the other hand, control methods of general nonlinear systems have been extensively developed since early 1980's, for example based on differential geometry theory (Isidori, 1995). Recently, the passivity approach has generated increasing interest for synthesizing control laws for nonlinear systems (Byrnes *et al.*, 1991), (Lin, 1995), (Jiang and Hill, 1998). An important problem in this field is how to achieve robust nonlinear control in the presence of unmodelled dynamics and external disturbances; along the same line there is the so-called H^∞ nonlinear control (Knobloch *et al.*, 1993), (Basar and Bernhard, 1995). It was noticed that one major difficulty with this approach,

alongside its possible system structural instability issue, seems to be caused by the requirement of solving the associated PDE equations. In order to alleviate this computational problem, the so-called inverse optimal control technique was recently developed based on the input-to-state stability (iss) concept (Krstic and Deng, 1998). This last reference extends previous inverse optimal control results, (Freeman and Kokotovic, 1996) and (Sepulchre *et al.*, 1997), for nonlinear systems with input uncertainties, to the case of disturbance attenuation of nonlinear systems affine in the disturbance.

In this paper, the aforementioned inverse optimal control technique is employed to further develop a very simple new control law for stabilization of the chaotic Chen's system, which in principle works for other complex nonlinear dynamical system as well. Computer simulation is also given for the purposes of illustration and verification.

II. SYSTEM DESCRIPTION

A new chaotic system, referred to as Chen's system by other authors, has recently been discovered (Chen and Ueta, 1999). This system is described by

$$\begin{aligned}\dot{x} &= a(y - x) \\ \dot{y} &= (c - a)x - xz + cy \\ \dot{z} &= xy - bz\end{aligned}\quad (1)$$

which has a chaotic attractor as shown in Fig. 1 when $a = 35$, $b = 3$, and $c = 28$. It has been experienced that this chaotic system is relatively difficult to control as compared to the Lorenz and Chua's system due to the prominent three-dimensional and complex topological features of its attractor.

We are interested in global asymptotically stabilizing this system to one of its unstable equilibrium points, $(0, 0, 0)$. Henceforth, we add a control input to the second state, so that the controlled system becomes