EXACT TRAVELLING WAVE SOLUTIONS TO THE GENERALIZED KURAMOTO-SIVASHINSKY EQUATION

Changpin Li†, Guanrong Chen‡ and Suchuan Zhao§

†*Department of Mathematics, Shanghai University, Shanghai 200436, P. R. China*
leecp@online.sh.cn
‡*Department of Electronic Engineering, City University of Hong Kong, Hong Kong, P. R. China*
gchen@ee.cityu.edu.hk
§*Department of Physics, Shanghai University, Shanghai 200436, P. R. China*

***Abstract*— By using a special transformation, the new exact travelling wave solutions to the generalized Kuramoto- Sivashinsky equation are obtained.**

***Keywords*— travelling wave solutions, solitary wave solutions, Kuramoto-Sivashinsky equation.**

I. INTRODUCTION

In this paper, we consider the generalized Kuramoto-Sivashinsky equation (Yang, 1994):

$$u_t + \beta u^\alpha u_x + \gamma u^\tau u_{xx} + \delta u_{xxxx} = 0\,, \tag{1}$$

where $\alpha, \beta, \gamma, \delta, \tau \in R$ and $\alpha\beta\gamma\delta \neq \mathbf{0}$.

When $\alpha = \beta = 1$ and $\tau = 0$, (1) reduces to the original Kuramoto-Sivashinsky (K-S) equation. The K-S equation was derived by Kuramoto (1978) for the study of phase turbulence in the Belousov-Zhabotinsky reaction. An extension of this equation to two or more spatial dimensions was then given by Sivashinsky (1977, 1980) in the study of the propagation of a frame front for the case of mild combustion. The K-S equation represents one class of pattern formation equation (Yang, 1994; Temam, 1988), and it also serves as a good model of bifurcation and chaos (Abdel-Gawad & Abdusalam, 2001; Li and Chen 2001, 2002).

As far as the travelling wave solutions are concerned, one can always use the transform

$$u(x,t) = u(\xi)\,, \quad \xi = x - ct\,, \tag{2}$$

where c is the wave velocity. The travelling wave solutions of (1) satisfy the following ordinary differential equation:

$$-cu' + \beta u^\alpha u' + \gamma u^\tau u'' + \delta u'''' = 0\,. \tag{3}$$

In (Yang, 1994), using the ansatz (Bernoulli equation)

$$u' = au + bu^n\,, \tag{4}$$

where $a, b, n \in R, ab < 0$ and $n \neq 1$, the exact travelling wave solution to (1) for $\alpha = 3\tau = 9$ was obtained. In this presentation, we further introduce the following ansatz:

$$u(\xi) = v^h(\xi)\,, \quad v' = av + bv^n\,, \tag{5}$$

where $\mathbf{abh \neq 0, n \neq 1}$ and $\mathbf{ab < 0}$, and obtain a new exact solution for the equation.

From (5), one first gets

$$u(\xi) = \left[-\frac{a}{2b}\tanh\left(\frac{n-1}{2}a(\xi - c_0)\right) - \frac{a}{2b}\right]^{\frac{h}{n-1}}\,, \tag{6}$$

in which c_0 is an arbitrary constant. If $h/(n-1) > 0$, (6) is the solitary wave solution connecting the two stationary states $u = 0$ and $u = (-\frac{a}{b})^{h/(n-1)}$ (Lu, *et al.*, 1993). So, the relative orbit is a heteroclinic orbit.

Repeating some differential calculations, one can obtain the following formulas:

$$v'' = (a + nbv^{n-1})v'\,, \tag{7}$$

$$v'''' = [a^3 + a^2bn(n^2+n+1)v^{n-1} + 3ab^2n^2(2n-1)v^{2n-2} + b^3n(2n-1)(3n-2)v^{3n-3}]v'\,. \tag{8}$$

$$u' = hv^{h-1}v'\,, \tag{9}$$

$$u'' = [h^2av^{h-1} + hb(n+h-1)v^{n+h-2}]v'\,, \tag{10}$$

$$u'''' = \{h^4a^3v^{h-1} + ha^2b(n+h-1)[h^2 + (n+h-1)\cdot(n+2h-1)]v^{n+h-2} + 3hab^2(n+h-1)^2(2n+h-2)v^{2n+h-3} + hb^3(n+h-1)(2n+h-2)(3n+h-3)v^{3n+h-4}\}\, v'\,. \tag{11}$$

Then, by substituting the first formula of (5) and (9)-(11) into (3), one has

$$\{(-ch + \delta h^4a^3)v^{h-1} + \beta hv^{\alpha h+h-1} + \gamma h^2av^{\tau h+h-1} + \gamma hb(n+h-1)v^{n+(\tau+1)h-2} + \delta ha^2b(n+h-1)\cdot [3h^2 + 3h(n-1) + (n-1)^2]v^{n+h-2} + 3\delta hab^2(n+h-1)^2\cdot (2n+h-2)v^{2n+h-3} + \delta hb^3(n+h-1)(2n+h-2)\cdot$$